QS026/1
**Matematik**
**Kertas 1**
**Semester II**
**Sesi 2003/2004**
**2 jam**
*Mathematics*
*Paper 1*
*Semester II*
*Session 2003/2004*
*2 hours*

**BAHAGIAN MATRIKULASI**
**KEMENTERIAN PENDIDIKAN MALAYSIA**
*MATRICULATION DIVISION*
*MINISTRY OF EDUCATION MALAYSIA*

**PEPERIKSAAN SEMESTER PROGRAM MATRIKULASI**
*SEMESTER EXAMINATION FOR MATRICULATION PROGRAMME*
**SEMESTER II**
**SESI 2003/2004**
*SEMESTER II*
*SESSION 2003/2004*

# MATEMATIK
**Kertas 1**
**2 jam**
*MATHEMATICS*
*Paper 1*
*2 hours*

**JANGAN BUKA KERTAS SOALAN INI SEHINGGA DIBERITAHU.**
*DO NOT OPEN THIS QUESTION BOOKLET UNTIL YOU ARE TOLD TO DO SO.*



Kertas soalan ini mengandungi **16** halaman bercetak.
This question booklet consists of **16** printed pages.

**INSTRUCTIONS TO CANDIDATE :**

This question booklet consists of **10** questions.

Answer **all** the questions.

The full marks for each question or section is shown in the bracket at the end of the question or section.

All steps must be clearly shown.

Only non–programmable scientific calculators can be used.

Numerical answers can be given in the form of $\pi$, e, surd, fractions or up to three significant figures, where appropiate, unless stated otherwise in the question.

# LIST OF MATHEMATICAL FORMULAE

## Trigonometry

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

## Differentiation and Intergration

| $f(x)$ | $f'(x)$ |
|---|---|
| $\cot x$ | $-\csc^2 x$ |
| $\sec x$ | $\sec x \tan x$ |
| $\csc x$ | $-\csc x \cot x$ |

$$\int \frac{f'(x)}{f(x)}\,dx = \ln|f(x)| + c$$

## Coordinate Geometry

Perpendicular distance from the point $(x_1, y_1)$ to the line $ax + bx + c = 0$ is

$$d = \frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$$

## Trapezium Rule

$$\int_a^b f(x)\,dx = \frac{h}{2}\{(y_0 + y_n) + 2(y_1 + y_2 + \ldots + y_{n-1})\}, \text{ where } h = \frac{b - a}{n}$$

## Newton–Raphson Method

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}, \quad n = 1, 2, 3, \ldots$$

| | | |
|---|---|---|
| **Sphere** | $V = \frac{4}{3}\pi r^3$ | $S = 4\pi r^2$ |
| **Right Circular Cone** | $V = \frac{1}{3}\pi r^2 h$ | $S = \pi rs$ |
| **Right Circular Cylinder** | $V = \pi r^2 h$ | $S = 2\pi rh$ |

1. A straight line $x-2y=0$ intersects a circle $x^2+y^2-8x+6y-15=0$ at the points $P$ and $Q$. Find the coordinate of $P$ and $Q$. Hence, find the equation of the circle passing through $P$, $Q$ and the point (1, 1) [6 marks]

2. Find the shortest distance between the lines $4x+3y+12=0$ and $4x+3y-15=0$. Determine the equation of the line equidistant between the two lines above. [5 marks]

3. Using the Newton-Raphson method and by taking $x_0 = 0.6$, estimate the root of the equation $x^3-3x^2+1=0$ correct to three decimal place. [6 marks]

4. Determine the general solution of the differential equation $x\frac{dy}{dx}-3y=x^3$.

   Hence, find the particular solution of the equation if it has a stationary point corresponding to $x = 1$. [7 marks]

5. Given an equation $4x^2-3y^2-8x+18y-59=0$.

   a) Show that the equation represents a hyperbola. [4 marks]

   b) Determine the coordinates at the center and vertices of the hyperbola. [3 marks]

   c) Find the slopes of the asymptotes. [1 marks]

   d) Sketch the hyperbola and label its asymptotes. [3 marks]

6. a) Find $\frac{dy}{dx}$ for each of the following cases:

i. $y=\cos(2-3x)$. [2 marks]

ii. $y=x^2 \sin 2x$. [2 marks]

iii. $y+\cos y = x+\cos x$. [2 marks]

b) Given the function $f(x) = 2x^3 - 6x^2 - 14x + 2$,

i. Determine the points on the graph of $f(x)$ where the slope is 4.

[4 marks]

ii. Find the normal line equation at the point obtained in (i) for $x > 0$

[2 marks]

7. Given two vectors $\mathbf{a}=2\mathbf{i}+\mathbf{j}+q\mathbf{k}$ and $\mathbf{b}=q\mathbf{i}-2\mathbf{j}+2q\mathbf{k}$.

a) Determine the values of $q$ such that **a** and **b** has the same magnitude. [4 marks]

b) If $q = 4$, find the angle between the vectors **b** and $\frac{1}{2}\left(\mathbf{a}-\frac{1}{2}\mathbf{b}\right)$. [3 marks]

c) i. Find the value of $q$ if $\mathbf{a}\times\mathbf{b} = 8\mathbf{i} - 4\mathbf{j} + 2\mathbf{k}$. [3 marks]

ii. Determine the Cartesian equation of the plane passing through point ( 1, 0, 2 ) and perpendicular to **a** and **b.** [3 marks]

8. Evaluate each of the following integrals:

a) $\int_0^{\frac{1}{2}\pi} \sin 5x \cos x \, dx.$ [4 marks]

b) $\int \frac{\cos^3 \theta}{\sin^2 \theta} d\theta.$ [4 marks]

c) $\int_0^{\frac{1}{3}\pi} \sin^2 \theta \cos^2 \theta \, d\theta.$ [4 marks]

9. Given that $f(\theta) = 4\cos\theta + 3\sin\theta$.

a) Express $f(\theta)$ in the form of $r\cos(\theta - \alpha)$ for $r > 0$ and $0° < \alpha < 90°$. [4 marks]

b) Sketch the curve for $f(\theta)$. [2 marks]

c) Find all the values of $\theta$ which satisfy $f(\theta) = 2$. [3 marks]

d) Find all the values of $\theta$ which satisfy $|f(\theta)| \leq 2$. [4 marks]

10. a) Given that $f(x) = x^4 - 2x^2 + 1$.

i. Determine the intervals of $x$-values where $f(x)$ is increasing and decreasing. [5 marks]

ii. Use the first derivative test to determine the local extremum of $f(x)$. [4 marks]

Based on the information obtained above, sketch the graph of $f(x)$.

b) A ladder 5 meter long is leaning against a vertical wall and is sliding down the wall. When the foot of the ladder is 3 meters away from the wall, the top of the ladder slides down at the rate of 0.5 m/s. At what rate is the foot of the ladder moving away from the wall at this instant. [6 marks]

**END OF QUESTION PAPER**