

QS026/1
Mathematics
Paper 1
Semester II
2009/2010
2 hours

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Matematik
Kertas 1
Semester II
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2 jam



BAHAGIAN MATRIKULASI
KEMENTERIAN PELAJARAN MALAYSIA
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PEPERIKSAAN SEMESTER PROGRAM MATRIKULASI
MATRICULATION PROGRAMME EXAMINATION

MATEMATIK
Kertas 1
2 jam

JANGAN BUKA KERTAS SOALAN INI SEHINGGA DIBERITAHU.
DO NOT OPEN THIS BOOKLET UNTIL YOU ARE TOLD TO DO SO.

CHOW CHOON WOUI

Kertas soalan ini mengandungi **13** halaman bercetak.

This booklet consists of 13 printed pages.

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INSTRUCTIONS TO CANDIDATE:

This question booklet consists of **10** questions.

Answer **all** questions.

The full marks for each question or section are shown in the bracket at the end of the question or section.

All steps must be shown clearly.

Only non-programmable scientific calculators can be used.

Numerical answers may be given in the form of π , e , surd, fractions or up to three significant figures, where appropriate, unless stated otherwise in the question.

LIST OF MATHEMATICAL FORMULAE

Trigonometry

$$\begin{aligned}\sin(A \pm B) &= \sin A \cos B \pm \cos A \sin B \\ \cos(A \pm B) &= \cos A \cos B \mp \sin A \sin B \\ \tan(A \pm B) &= \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}\end{aligned}$$

$$\begin{aligned}\sin A + \sin B &= 2 \sin \frac{A+B}{2} \cos \frac{A-B}{2} \\ \sin A - \sin B &= 2 \cos \frac{A+B}{2} \sin \frac{A-B}{2} \\ \cos A + \cos B &= 2 \cos \frac{A+B}{2} \cos \frac{A-B}{2} \\ \cos A - \cos B &= -2 \sin \frac{A+B}{2} \sin \frac{A-B}{2}\end{aligned}$$

Limit

$$\begin{aligned}\lim_{h \rightarrow 0} \frac{\sin h}{h} &= 1 \\ \lim_{h \rightarrow 0} \frac{1 - \cos h}{h} &= 0\end{aligned}$$

Hyperbolic

$$\begin{aligned}\sinh(x+y) &= \sinh x \cosh y + \cosh x \sinh y \\ \cosh(x+y) &= \cosh x \cosh y + \sinh x \sinh y \\ \cosh^2 x - \sinh^2 x &= 1 \\ 1 - \tanh^2 x &= \operatorname{sech}^2 x \\ \coth^2 x - 1 &= \operatorname{cosech}^2 x \\ \sinh 2x &= 2 \sinh x \cosh x \\ \cosh 2x &= \cosh^2 x + \sinh^2 x\end{aligned}$$

LIST OF MATHEMATICAL FORMULAE

Differentiation and Integration

$f(x)$	$f'(x)$
$\cot x$	$-\operatorname{cosec}^2 x$
$\sec x$	$\sec x \tan x$
$\operatorname{cosec} x$	$-\operatorname{cosec} x \cot x$

$\operatorname{coth} x$	$-\operatorname{cosech}^2 x$
$\operatorname{sech} x$	$-\operatorname{sech} x \tanh x$
$\operatorname{cosech} x$	$-\operatorname{cosech} x \operatorname{coth} x$

$$\int \frac{f'(x)}{f(x)} dx = \ln |f(x)| + c$$

$$\int u dv = uv - \int v du$$

Sphere

$$V = \frac{4}{3} \pi r^3$$

$$S = 4 \pi r^2$$

Right Circular Cone

$$V = \frac{1}{3} \pi r^2 h$$

$$S = \pi r s$$

Right circular cylinder

$$V = \pi r^2 h$$

$$S = 2 \pi r h$$

1 An ellipse with centre at the origin passes through the points $(0, 3)$ and $(1, 1)$.
Find the equation and the foci of the ellipse. [7 marks]

2 Prove that $1 + \tan x \tan 2x = \sec 2x$. [5 marks]

3 Given $\underline{u} = 2\underline{i} - 2\underline{j} + \underline{k}$. Find the vectors which have magnitude 6 and parallel to \underline{u} . [6 marks]

4 Using the definition of hyperbolic functions, prove that

$$\frac{1}{\cosh x + \sinh x} = \cosh x - \sinh x.$$

Hence, find

$$\int \frac{1}{(\cosh x + \sinh x)^2} dx. \quad [7 \text{ marks}]$$

5 Given that $f(x) = \frac{x^3 + 8}{x}$.

(a) State the asymptote of f . [1 mark]

(b) Find the critical and inflection points of f . [5 marks]

(c) Determine the intervals where f is increasing and f is decreasing. [3 marks]

(d) Sketch the graph of f . [3 marks]

- 6 (a) Prove that for $\theta \neq n\pi$, where n is an integer,

$$\frac{\sin \theta}{\cos \theta (1 - 2 \sin^2 \theta + \sin^4 \theta)} = \tan \theta \sec^4 \theta. \quad [3 \text{ marks}]$$

- (b) By using (a) and the substitution $u = \tan \theta$, evaluate

$$\int_0^{\pi/4} \frac{\sin \theta}{\cos \theta (1 - 2 \sin^2 \theta + \sin^4 \theta)} d\theta. \quad [8 \text{ marks}]$$

- 7 The points $A(1, 3, 2)$, $B(3, -1, 6)$ and $C(5, 2, 0)$ lie on the plane Π . A line L passes through the points $P(1, 2, 2)$ and $Q(0, 1, 4)$. Find

- (a) $\overrightarrow{AB} \times \overrightarrow{AC}$ and hence, obtain an equation of the plane Π in Cartesian form. [7 marks]
- (b) the parametric equations of the line L . [3 marks]
- (a) the point of intersection of L and Π . [3 marks]

- 8 (a) If $\cos(x + y) = 2x \sin y$ where $0 \leq y \leq \pi$, find y and $\frac{dy}{dx}$ at $x = 0$.

[6 marks]

- (b) Given $y = \sin(\ln x)$, show that $x^2 \frac{d^2 y}{dx^2} + x \frac{dy}{dx} + y = 0$.

[6 marks]

- 9 (a) The position of a particle moving along a straight line at any time $t > 0$ is given by $s(t) = t(t-1)(t-2)$, where s is the distance of the particle from the origin. Find the velocity of the particle at the instant when the acceleration becomes zero. [4 marks]

- (b) A closed right circular cylindrical container of radius r and height h is to be constructed with volume $4,000 \text{ cm}^3$. The cost for the construction is RM 1.00 per cm^2 for the curved surface while RM 2.00 per cm^2 for the top and bottom surfaces. State h in terms of r and hence, find the radius of the cylinder so that the cost of the construction is minimum. [8 marks]

- 10 Given the circles

$$C_1 : x^2 + y^2 - 2x - 2y + 1 = 0$$

$$C_2 : x^2 + y^2 = 1.$$

Find

- (a) the centre and the radius of the circle C_1 . [3 marks]
- (b) the equations of the tangents from the point $(0, 3)$ to the circle C_2 . [5 marks]
- (c) the equation of the circle that passes through the point $(-5, 0)$ and the points of intersection of the circles C_1 and C_2 . [7 marks]

END OF BOOKLET