QM016/2
Mathematics
Paper 2
Semester I
Session 2004/2005
2 hours

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Matematik
Kertas 2
Semester I
Sesi 2004/2005
2 jam

BAHAGIAN MATRIKULASI
KEMENTERIAN PELAJARAN MALAYSIA
MATRICULATION DIVISION
MINISTRY OF EDUCATION MALAYSIA

PEPERIKSAAN SEMESTER PROGRAM MATRIKULASI
MATRICULATION PROGRAMME EXAMINATION

MATEMATIK
Kertas 2
2 jam

JANGAN BUKA KERTAS SOALAN INI SEHINGGA DIBERITAHU.
DO NOT OPEN THIS BOOKLET UNTIL YOU ARE TOLD TO DO SO.

INSTRUCTIONS TO CANDIDATE:

This question booklet consists of **10** questions.

Answer **all** questions.

The full marks are shown in the brackets at the end of each question or section.

All work must be clearly shown.

The usage of electronic calculator is allowed.

Numerical answers can be given in the form of π, e, surd, fractions or up to three significant figures, where appropriate, unless otherwise stated in the question.

LIST OF MATHEMATICAL FORMULAE

Differentiation

If $y = g(t)$ and $x = f(t)$, then $\dfrac{dy}{dx} = \dfrac{dy}{dt} \div \dfrac{dx}{dt}$.

Integration

$$\int u\,dv = uv - \int v\,du$$

1. Given $g(x) = \sqrt[3]{x}$ and $h(x) = \dfrac{1}{x^3}$.

    (a)   Find $f(x)$ such that $(f \circ g \circ h)(x) = \dfrac{x}{x+1}$.   [3 marks]

    (b)   Determine the domain of $(f \circ g \circ h)(x)$.   [3 marks]

2. Find each of the following limits, if they exist.

    (a) $\displaystyle\lim_{x \to 0}\left(\dfrac{e^{3x}-1}{e^x-1}\right)$,   [3 marks]

    (b) $\displaystyle\lim_{x \to +\infty}\dfrac{\sqrt{x^2-2}}{3x+6}$ [3 marks]

3. If $xy = 2(x-y)^2$, find the following values at the point $(1, 2)$.

    (a) $\dfrac{dy}{dx}$ [3 marks]

    (b) $\dfrac{d^2y}{dx^2}$ [4 marks]

4. By substituting $u^2 = x+1$, determine $\displaystyle\int \dfrac{1}{x\sqrt{x+1}}\, dx$.   [7 marks]

5. Find $\displaystyle\int x^2 e^{2x}\, dx$.   [7 marks]

    Hence, find the volume of the solid generated by the area bounded by the curve $y = xe^x$ and the lines $x = 0$, $x = 1$, $y = 0$ which is rotated at 2π radian about the x-axis.   [4 marks]

6. Express $\dfrac{x^2-2x-9}{(2x-1)(x^2+3)}$ as partial fractions. [6 marks]

Hence, evaluate

$$\int_1^2 \dfrac{x^2-2x-9}{(2x-1)(x^2+3)}\,dx,$$

giving the answer correct to three significant figures. [5 marks]

7. (a) Given $y=2^{x^2}$, find $\dfrac{dy}{dx}$. [4 marks]

(b) If $y=e^x\ln(1+x)$, show that

$$(x+1)^2\left(\dfrac{d^2y}{dx^2}-\dfrac{dy}{dx}\right)=xe^x$$

[8 marks]

8. Given $x=2t-\dfrac{1}{t}$ and $y=t+\dfrac{4}{t}$, where t is a non-zero parameter.

(a) Show that

$$\dfrac{dy}{dx}=\dfrac{1}{2}\left(1-\dfrac{9}{2t^2+1}\right).$$

[6 marks]

(b) Hence deduce that $\dfrac{dy}{dx}<\dfrac{1}{2}$ for all t. [2 marks]

(c) Find $\dfrac{d^2y}{dx^2}$ when $t=1$. [4 marks]

9. (a) State the conditions for the function f to be continuous at $x = c$. [1 mark]

    (b) Given that

    $$q(x) = \begin{cases} \dfrac{x^2 - 4}{x - 2}, & x \neq 2 \\ \\ 3, & x = 2. \end{cases}$$

    (i) Sketch the graph of q. [2 marks]

    (ii) Discuss the continuity of q at $x = 2$. [4 marks]

    (c) Determine the values of A and B such that

    $$f(x) = \begin{cases} x, & x \leq 1 \\ Ax + B, & 1 < x < 4 \\ -2x, & x \geq 4 \end{cases}$$

    is continuous on the interval $(-\infty, \infty)$. [6 marks]

10. A function f is defined by

    $$f(x) = \frac{x}{\sqrt{x^2 - 9}}.$$

    (a) State the domain of f. [1 mark]

    (b) Find the vertical asymptotes. [1 mark]

    (c) Determine $\lim\limits_{x \to \infty} f(x)$ and $\lim\limits_{x \to -\infty} f(x)$. Hence, state the horizontal asymptotes. [4 marks]

    (d) Find f^{-1} and determine the range of f. [5 marks]

    (e) Sketch the graph of f. [4 marks]

END OF QUESTION BOOKLET

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