

QM016/1
Mathematics
Paper 1
Semester I
Session 2004/2005
2 hours

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Matematik
Kertas 1
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2 jam



BAHAGIAN MATRIKULASI
KEMENTERIAN PELAJARAN MALAYSIA
MATRICULATION DIVISION
MINISTRY OF EDUCATION MALAYSIA

PEPERIKSAAN SEMESTER PROGRAM MATRIKULASI
MATRICULATION PROGRAMME EXAMINATION

MATEMATIK
Kertas 1
2 jam

JANGAN BUKA KERTAS SOALAN INI SEHINGGA DIBERITAHU.
DO NOT OPEN THIS BOOKLET UNTIL YOU ARE TOLD TO DO SO.

CHOW CHOON WOOL

Kertas soalan ini mengandungi 11 halaman bercetak.
This booklet consists of 11 printed pages.

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INSTRUCTIONS TO CANDIDATE:

This question booklet consists of **10** questions.

Answer **all** questions.

The full marks are shown in the brackets at the end of each question or section.

All work must be clearly shown.

The usage of electronic calculator is allowed.

Numerical answers can be given in the form of π , e , surd, fractions or up to three significant figures, where appropriate, unless otherwise stated in the question.

LIST OF MATHEMATICAL FORMULAE

For the quadratic equation $ax^2 + bx + c = 0$:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

For an arithmetic series:

$$T_n = a + (n-1)d$$

$$S_n = \frac{n}{2} [2a + (n-1)d]$$

For a geometric series

$$T_n = ar^{n-1}$$

$$S_n = \frac{a(1-r^n)}{1-r}, \quad r \neq 1$$

Binomial Expansion:

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \dots + \binom{n}{r} a^{n-r} b^r + \dots + b^n,$$

$$\text{where } n \in \mathbb{N} \text{ and } \binom{n}{r} = \frac{n!}{(n-r)! r!}.$$

1. Solve $x^5 e^{-3 \ln x} + 4x = 21$. [5 marks]
2. Express $5.555\dots$ in the form of a geometric series. Hence find the
- (a) sum of the first n terms [4 marks]
- (b) infinite sum [2 marks]
- of the series.
3. Using mathematical induction method, show that
- $$(1)(3) + (2)(4) + (3)(5) + \dots + n(n+2) = \frac{n}{6}(n+1)(2n+7).$$
- [6 marks]
4. Given $z_1 = 1 - 3i$ and $z_2 = 2 + 5i$.
- (a) Express $\frac{1}{z_1} + \frac{1}{z_2}$ in the form of $a + bi$. [3 marks]
- (b) Find the argument of \bar{z}_2 in radian. [3 marks]
5. (a) Given $(x+3)$ is one factor of $P(x) = 9 - 12x - 11x^2 - 2x^3$. Factorise completely $P(x)$, and express $\frac{13x+18}{P(x)}$ as a sum of partial fractions. [8 marks]
- (b) Expand $\frac{-3}{(x+3)^2}$ in increasing power up to the fourth term. [3 marks]

6. Solve the following inequalities:

(a) $4x + \frac{9}{x} \geq 12$ [3 marks]

(b) $1 + \log_2 x - 6\log_x 2 > 0$ [4 marks]

(c) $\left| \frac{x+5}{2x-4} \right| \geq 1$ [7 marks]

7. Show that

(a) $\sum_{i=1}^r (a_{i+1} - a_i) = a_{r+1} - a_1$. [3 marks]

(b) $a_{i+1} - a_i = 2i3^i + 3^{i+1}$ if $a_i = i3^i$. Hence, using (a) evaluate $\sum_{i=1}^r 2i3^i$ in terms of r . [10 marks]

8. A polynomial has the form $P(x) = 2x^3 - 3x^2 - px + q$, with x real and p, q constants.

(a) When $P(x)$ is divided by $(x-1)$ the remainder is $(2-4x)$. Find the values of p and q , and factorize $P(x)$ completely if 2 is one of the roots. [7 marks]

(b) Hence, form a quadratic equation $ay^2 + by + c = 0$ if the sum of roots is the sum of all the roots of $P(x)$ and the product of roots is the product of all the roots of $P(x)$. Also, find the roots of the quadratic equation that is formed. [5 marks]

9. (a) Given $z = x + yi$, where x and y are real numbers. If $\left| \frac{z+i}{z+1+i} \right| = 2$, show that $3x^2 + 3y^2 + 8x + 6y + 7 = 0$. [5 marks]

(b) Given $x = 3 + 4i$ is one root of $P(x) = 2x^3 - 9x^2 + 32x + 75$, find all the roots of $P(x)$. [7 marks]

10. (a) Show that the determinant of the matrix $A = \begin{pmatrix} x^2 & x & 1 \\ y^2 & y & 1 \\ z^2 & z & 1 \end{pmatrix}$

is $(y-x)(z-x)(y-z)$ for real x, y and z . [3 marks]

(b) By substituting $x = 1, y = 2$ and $z = 3$, the matrix A becomes $\begin{pmatrix} 1 & 1 & 1 \\ 4 & 2 & 1 \\ 9 & 3 & 1 \end{pmatrix}$.

Find the adjoint and inverse of the matrix A . [6 marks]

(c) The graph of a quadratic equation $y = ax^2 + bx + c$ passes through the points whose coordinates are $(1, 2), (2, 3)$ and $(3, 6)$.

(i) Obtain a system of linear equations to represent the given information. [2 marks]

(ii) Write down the system in (i) as a matrix equation in the form of

$$Ax = b \text{ where } x = \begin{pmatrix} a \\ b \\ c \end{pmatrix}. \quad [1 \text{ mark}]$$

(iii) Use the inverse of the matrix to solve the system of linear equations in (ii). Hence, find the quadratic equation of the graph. [3 marks]

END OF QUESTION BOOKLET