BAHAGIAN MATRIKULASI
KEMENTERIAN PELAJARAN MALAYSIA
MATRICULATION DIVISION
MINISTRY OF EDUCATION MALAYSIA

PEPERIKSAAN SEMESTER PROGRAM MATRIKULASI
MATRICULATION PROGRAMME EXAMINATION

MATEMATIK
Kertas 2
2 jam

JANGAN BUKA KERTAS SOALAN INI SEHINGGA DIBERITAHU.
DO NOT OPEN THIS BOOKLET UNTIL YOU ARE TOLD TO DO SO.

INSTRUCTIONS TO CANDIDATE:

This question booklet consists of **10** questions.

Answer **all** questions.

The full marks for each question or section are shown in the bracket at the end of each of the question or section.

All steps must be shown clearly.

Only non-programmable scientific calculators can be used.

Numerical answers can be given in the form of π, e, surd, fractions or up to three significant figures, where appropriate, unless stated otherwise in the question.

LIST OF MATHEMATICAL FORMULAE

Differentiation

If $y = g(t)$ dan $x = f(t)$, maka $\dfrac{dy}{dx} = \dfrac{dy}{dt} \div \dfrac{dx}{dt}$.

Integration

$$\int u\,dv = uv - \int v\,du$$

1. Functions f and g are defined as

$$f(x) = e^{2x} \quad , \quad g(x) = 1 - x, \qquad x \in R.$$

Find $f^{-1}(x)$ and hence obtain $\left(g \circ f^{-1}\right)(x)$.

[5 marks]

2. Using integration by parts, evaluate $\displaystyle\int_0^1 xe^{-2x}dx$.

[6 marks]

3. By taking logarithm on both sides of the equation

$$y = \left(x^2\right)^{\sqrt{x}},$$

show that $y' = \dfrac{1}{\sqrt{x}}(\ln x + 2)\left(x^2\right)^{\sqrt{x}}$.

[6 marks]

4. Find the following limits.

(a) $\displaystyle\lim_{x \to 2^-} \frac{x^2 - 4}{|x - 2|}$.

[3 marks]

(b) $\displaystyle\lim_{x \to 0} \left(\frac{1}{x\sqrt{1+x}} - \frac{1}{x}\right)$.

[4 marks]

5. (a) State the conditions of continuity of a function f defined in the closed interval $[a, b]$. [2 marks]

    (b) If function f is defined by

    $$f(x) = \begin{cases} 1+x^2, & -3 \le x \le 0 \\ 2-x, & 0 < x \le 2 \\ (x-2)^2, & 2 < x \le 5 \end{cases}$$

    sketch its graph.

    Using the conditions of continuity in (a), determine the value of x where the function is not continuous. [5 marks]

    (c) If function g is defined as

    $$g(x) = \begin{cases} Ax+2, & -3 \le x \le 1 \\ x^2 + Bx + A, & 1 < x < 2 \\ 1 - \dfrac{1}{x}, & 2 \le x \le 5 \end{cases}$$

    find the values of A and B such that g is continuous in the interval $[-3, 5]$.

    [5 marks]

6. Given $y = Ax + \dfrac{B}{x^2}$, where A and B are constants and $x \ne 0$.

    (a) Find $\dfrac{dy}{dx}$ and $\dfrac{d^2y}{dx^2}$.

    Hence, show that $x^2 \dfrac{d^2y}{dx^2} + 2x \dfrac{dy}{dx} - 2y = 0$. [5 marks]

    (b) Find the values of A and B if $y = 3$ and $y' = -6$ when $x = 1$. [2 marks]

    (c) If $y = f(x)$, find $\lim\limits_{x \to 0} f(x)$, $\lim\limits_{x \to -\infty} f(x)$ and $\lim\limits_{x \to +\infty} f(x)$ where the values of A and B as found in (b). Hence, sketch the graph of f. [5 marks]

7. Express $\dfrac{5x^3 - x^2 + x - 1}{x^2(x^2 + 1)}$ in the form of partial fractions.

Hence, evaluate $\displaystyle\int_1^e \dfrac{5x^3 - x^2 + x - 1}{x^2(x^2 + 1)}\,dx$.

Give the answer correct to three significant figures. [12 marks]

8. (a) Evaluate $\displaystyle\int_1^2 x^3 \sqrt{x^4 + 5}\ dx$. [5 marks]

(b) Given $f(x) = \begin{cases} xe^{x^2}, & x < 0 \\ 3^{x+1}, & x \geq 0 \end{cases}$, find $\displaystyle\int_{-2}^2 f(x)\,dx$. [7 marks]

9. Consider the parametric equations,

$$x = t^2 \quad \text{and} \quad y = t^3 - 3t .$$

(a) Evaluate $\dfrac{dy}{dx}$ at the point $(3, 0)$. [5 marks]

(b) Find the point (x, y) where $\dfrac{dy}{dx}$ is not defined. [2 marks]

(c) Determine the interval of t such that $\dfrac{d^2 y}{dx^2} < 0$. [6 marks]

10.   Let $f(x) = \dfrac{\ln x}{x}, \quad 1 \le x \le e.$

   (a)   Find the area of the region bounded by $f(x)$ and the x-axis.   [5 marks]

   (b)   Hence, find the volume of the solid generated by revolving the region 2π radian

   about the x-axis. Give the answer in terms of e and π.   [10 marks]

END OF QUESTION BOOKLET