QM016/1
Mathematics
Paper 1
Semester I
2007/2008
2 hours

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Matematik
Kertas 1
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BAHAGIAN MATRIKULASI
KEMENTERIAN PELAJARAN MALAYSIA
MATRICULATION DIVISION
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PEPERIKSAAN SEMESTER PROGRAM MATRIKULASI
MATRICULATION PROGRAMME EXAMINATION

MATEMATIK
Kertas 1
2 jam

JANGAN BUKA KERTAS SOALAN INI SEHINGGA DIBERITAHU.
DO NOT OPEN THIS BOOKLET UNTIL YOU ARE TOLD TO DO SO.

Kertas soalan ini mengandungi **11** halaman bercetak.
This booklet consists of **11** *printed pages.*

INSTRUCTIONS TO CANDIDATE:

This question booklet consists of **10** questions.

Answer **all** questions.

The full marks allocated for each question or section is shown in the bracket at the end of each question or section.

All steps must be shown clearly.

Only non-programmable scientific calculator can be used.

Numerical answers can be given in the form of π, e, surd, fractions or correct to three significant figures, where appropriate, unless stated otherwise in the question.

LIST OF MATHEMATICAL FORMULAE

Arithmetic Series:

$$T_n = a + (n-1)d$$

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

Geometric Series:

$$T_n = ar^{n-1}$$

$$S_n = \frac{a(1-r^n)}{1-r} \quad \text{for } r < 1$$

Binomial Expansions:

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \cdots + \binom{n}{r}a^{n-r}b^r + \cdots + b^n$$, where $n \in \mathrm{N}$ and

$$\binom{n}{r} = \frac{n!}{r!(n-r)!}$$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \cdots + \frac{n(n-1)\cdots(n-r+1)}{r!}x^r + \cdots \quad \text{for } |x| < 1$$

1. Given that $81^{y} = 3^{(2y-3)x}$ and $2^{18y+6x} = 64^{xy}$. Find the values of x and y.

[6 marks]

2. Express $\dfrac{2x+1}{(x+2)(x^2-2x+4)}$ in partial fractions. **[6 marks]**

3. If $z_1 = 4 - i$ and $z_2 = 1 - 2i$, find $z_1 - \dfrac{5}{z_2}$. Express the answer in polar form.

[6 marks]

4. The sum of the first n terms of an arithmetic series is $\dfrac{n}{2}(3n-5)$. If the second and fourth terms of the arithmetic series are the second and the third terms of a geometric series respectively, find the sum of the first eleven terms of this geometric series.

[7 marks]

5. The quadratic equation $x^2 + k(x+2) - (x+6) = 0$ has roots α and β, where k is a constant.

(a) Find a quadratic equation with roots $\dfrac{1}{\alpha}$ and $\dfrac{1}{\beta}$ in terms of k.

[5 marks]

(b) Find $\alpha^2 + \beta^2$ in terms of k. Hence, determine the minimum value of $\alpha^2+\beta^2$.

[4 marks]

6. (a) Find a cubic polynomial $Q(x) = (x+a)(x+b)(x+c)$ satisfying the following conditions:

the coefficients of x^3 is 1, $Q(-1) = 0$, $Q(2) = 0$, and $Q(3) = -8$.

[4 marks]

(b) A polynomial $P(x) = ax^3 - 4x^2 + bx + 18$ has a factor $(x+2)$ and a remainder $(2x+18)$ when divided by $(x+1)$. Find the values of a and b. Hence, factorize $P(x)$ completely.

[8 marks]

7. Solve the following inequalities:

(a) $\dfrac{x}{x+4} \leq \dfrac{1}{2x-1}$. **[6 marks]**

(b) $\left|\dfrac{x}{x+4}\right| < 2$. **[7 marks]**

8. A system of linear equations is given as

$$\begin{aligned} ax - 2y - 3z &= b \\ 2x - y + 4z &= 2 \\ 4x + 3y - 2z &= 14 \end{aligned}$$

where a and b are constants.

(a) Find x and z in terms of a and b using Cramer's rule. **[9 marks]**

(b) Determine the conditions of a and b for which the above system

(i) has a unique solution.

(ii) has no solution.

[4 marks]

9. Given that $A = \begin{bmatrix} 1 & a & 2 \\ 2 & 1 & 2 \\ 2 & 2 & b \end{bmatrix}$, where a and b are constants.

(a) If $|A| = -13$, evaluate the determinant of matrix $\begin{bmatrix} 4 & 1 & 2 \\ 2 & a & 2 \\ 4 & 2 & b \end{bmatrix}$ using determinant properties.

[4 marks]

(b) Given that $A^2 - 4A = 5I$, where I is a 3×3 identity matrix. Show that $a = 2$ and $b = 1$. Hence, find A^{-1}.

[9 marks]

10. Given that $f(x)=\dfrac{1}{1+x}$, $x\neq -1$ and $g(x)=\dfrac{1}{2-x}$, $x\neq 2$.

(a) Expand $f(x)$ and $g(x)$ as a series of ascending powers of x up to the term containing x^n. Hence, estimate the value of $(1.9)^{-1}$ using the first four terms of $g(x)$.

[7 marks]

(b) If $h(x)=f(x)+g(x)$, show that the coefficient of x^n for $h(x)$ is $(-1)^n+\dfrac{1}{2^{n+1}}$. Hence, obtain the coefficient of x^3 for $h(x)$.

[5 marks]

(c) Find the coefficient of x^2 for $\dfrac{f(x)}{g(x)}$. **[3 marks]**

END OF QUESTION BOOKLET