

QM016/2  
Mathematics  
Paper 2  
Semester I  
2009/2010  
2 hours

QM016/2  
Matematik  
Kertas 2  
Semester I  
2009/2010  
2 jam



**BAHAGIAN MATRIKULASI**  
**KEMENTERIAN PELAJARAN MALAYSIA**  
*MATRICULATION DIVISION*  
*MINISTRY OF EDUCATION MALAYSIA*

**PEPERIKSAAN SEMESTER PROGRAM MATRIKULASI**  
*MATRICULATION PROGRAMME EXAMINATION*

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**MATEMATIK**  
**Kertas 2**  
**2 jam**

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**JANGAN BUKA KERTAS SOALAN INI SEHINGGA DIBERITAHU.**  
*DO NOT OPEN THIS BOOKLET UNTIL YOU ARE TOLD TO DO SO.*

CHOW CHOON WOOL

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Kertas soalan ini mengandungi **11** halaman bercetak.  
*This booklet consists of 11 printed pages.*

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**INSTRUCTIONS TO CANDIDATE:**

This question booklet consists of **10** questions.

Answer **all** questions.

The full marks for each question or section are shown in the bracket at the end of the question or section.

All steps must be shown clearly.

Only non-programmable scientific calculators can be used.

Numerical answers may be given in the form of  $\pi$ ,  $e$ , surd, fractions or up to three significant figures, where appropriate, unless stated otherwise in the question.

## LIST OF MATHEMATICAL FORMULAE

**Differentiation**

If  $y = g(t)$  and  $x = f(t)$ , then  $\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$

$$\frac{d^2y}{dx^2} = \frac{\frac{d}{dt} \left( \frac{dy}{dx} \right)}{\frac{dx}{dt}}$$

**Integration**

$$\int u dv = uv - \int v du$$

- 1 A function  $g$  is defined by

$$g(x) = \frac{1}{\sqrt{x-1}}, \quad x > 1.$$

Find  $g^{-1}(x)$  and state its domain and range.

[5 marks]

- 2 A function  $f$  is given as

$$f(x) = \begin{cases} |x+1|, & x < 0 \\ 2, & x = 0 \\ e^{-2x}, & x > 0. \end{cases}$$

Find  $\lim_{x \rightarrow 0^-} f(x)$ ,  $\lim_{x \rightarrow 0^+} f(x)$  and  $\lim_{x \rightarrow 0} f(x)$ .

Hence, determine whether  $f$  is continuous at  $x = 0$ . Give a reason to your answer.

[6 marks]

- 3 If  $y = x + e^x$ , show that  $\frac{d^2x}{dy^2} + \frac{e^x}{(1+e^x)^3} = 0$ .

[6 marks]

- 4 Evaluate  $\int_2^3 \frac{x+1}{x^2(x-1)} dx$ .

[7 marks]

- 5 A parametric curve is given by  $x = t - \frac{1}{t}$ ,  $y = t + \frac{1}{t}$ ,  $t \neq 0$ .

(a) Find  $\frac{dy}{dx}$  in terms of  $t$  and evaluate it at  $t = -2$ .

[4 marks]

(b) Find the value of  $\frac{d^2y}{dx^2}$  at  $t = 1$ , and evaluate  $\lim_{t \rightarrow \infty} \frac{d^2y}{dx^2}$ .

[8 marks]

- 6 (a) Show that  $y - \sqrt{y^2 + 1} < 0$  for all real values of  $y$ .

[2 marks]

- (b) Let  $f$  be a function defined by  $f(x) = \frac{e^x - e^{-x}}{2}$ . Find  $f^{-1}(x)$ .

[6 marks]

- (c) Evaluate  $\lim_{x \rightarrow 0} \frac{e^{2x} - e^{-2x}}{e^x - e^{-x}}$ .

[3 marks]

- 7 A function  $f$  is defined by

$$f(x) = \begin{cases} 34, & x = -4 \\ 0, & x = 2 \\ 17, & x = 4 \\ \frac{x^4 - 3x^2 - 4}{x^2 + x - 6}, & x \neq -4, x \neq -3, x \neq 2, x \neq 4 \end{cases}$$

- (a) Evaluate  $\lim_{x \rightarrow 2} f(x)$ .

[4 marks]

- (b) Find the interval(s) where  $f$  is continuous on the interval  $[-4, 4]$ .

[8 marks]

- 8 (a) Given a function  $g$  defined by

$$g(x) = \begin{cases} xe^{x^2}, & x \leq 1 \\ \frac{(\ln x)^2}{x}, & x > 1. \end{cases}$$

Evaluate  $\int_{-1}^3 g(x) dx$ .

[6 marks]

- (b) Use integration by parts to show that

$$\int \frac{xe^{2x}}{\sqrt{e^{2x} + 1}} dx = (x-1)\sqrt{e^{2x} + 1} - \int \frac{1}{\sqrt{e^{2x} + 1}} dx.$$

[7 marks]

- 9 (a) Let  $f$  and  $g$  be functions such that  $f(x) = x^2g(x^2)$  with  $g(1) = 2$  and  $g'(1) = 1$ . Find  $f'(1)$ .  
[4 marks]
- (b) Given a curve  $y = x + \frac{1}{x}$ .
- (i) Determine the gradient of the curve  $y = x + \frac{1}{x}$  at  $x = b$  in terms of  $b$ .
- (ii) Find the value of  $b$  if a straight line with the gradient in (i) passes through the points  $(b, b + \frac{1}{b})$  and  $(0, 4)$ .
- (iii) Hence, find the equation of a line perpendicular to the line in (ii) at  $(0, 4)$ .  
[9 marks]
- 10 A region  $R$  is bounded by the curve  $y = x(x - 2)$  and line  $y = x$ .
- (a) Sketch the graphs and shade the region  $R$ .  
[2 marks]
- (b) Find the area of  $R$ .  
[3 marks]
- (c) Find the volume of the solid obtained when the part of  $R$  above the  $x$ -axis is rotated through  $360^\circ$  about the  $x$ -axis.  
[5 marks]
- (d) Let  $R$  forms the surface of water in a pond where the depth of the water at any point  $(x, y)$  in  $R$  is given by  $x + 5$ . Find the volume of the water in the pond.  
[5 marks]

END OF QUESTION BOOKLET

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