KEMENTERIAN
PENDIDIKAN
MALAYSIA

SM025/2

Matriculation Programme Examination

Semester 2

Session 2018/2019

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1. a) Evaluate $\int x(1-x)^8\,dx$ by using a suitable substitution.

   b) Find the area of the region bounded by the curve $y = x\cos x$ and $x - axis$ between $x = 0$ and $x = \frac{\pi}{2}$. Give your solution in term of π.

2. Solve $\frac{dy}{dx} + 2(x+1)y^2 = 0$, given that $y = 1$ when $x = 1$. Express y in terms of x.

3. Use Newton-Raphson method to solve the equation $e^x - x - 2 = 0$ correct to four decimal places by taking $x_1 = 1$ as the first approximation.

4. An ellipse $Ax^2 + y^2 + Bx + Cy + 1 = 0$ passes through points $(0,1), (1,-1)$ and $(2,1)$.
   a) Find the equation of the ellipse in the standard form. Hence, state the centre and vertices of the ellipse.
   b) Find the foci of th ellipse.
   c) Sketch the graph of the ellipse.

5. The line L_1 and L_2 passes through the point $R(2,4,-3)$ and $S(8,-5,9)$in the direction of $2\boldsymbol{i} - 3\boldsymbol{j} + 4\boldsymbol{k}$ and $\boldsymbol{i} - 2\boldsymbol{j} + 3\boldsymbol{k}$, respectively.
   a) State the equations for lines L_1 and L_2 in the vector form. Hence, calculate the acute angle between the lines L_1 and L_2.
   b) Find the equation of plane containing the line L_1 and the point $(7,-3,5)$ in the Cartesian form.
   c) Determine whether the line L_2 is parallel to the plane $x + 5y + 3z = 5$.

6. **TABLE 1** shows the frequency distribution of the diameter of 100 pebbles which are measured to the nearest millimeter(mm).

TABLE 1

Diameter (mm)	Frequency
10 – 14	22
15 – 19	20
20 – 24	25
25 – 29	15
30 - 34	18

   Calculate the
   a) mean.
   b) mode.
   c) median.

7. A committee of 5 is to be selected from a group of 7 men and 6 women. How many different committees could be formed if
   a) there is no woman in the committee?
   b) a particular man must be in the committee and the remaining has equal number of men and women?
   c) at least 3 men are in the committee?

8. The probabilities of events X and Y are given as $P(X) = \frac{3}{5}$, $P(X'|Y) = \frac{31}{45}$ and $P(X \cap Y) = \frac{2}{25}$.
   a) Show that $P(Y) = \frac{9}{35}$
   b) Find $P(X \cup Y')$.

9. A discrete random variable X has the probability distribution function

$$f(x) = \begin{cases} \dfrac{x+1}{16}, & x = 2, 3, 4 \\ kx, & x = 6, 8 \\ 0, & otherwise \end{cases}$$

   a) Show that $k = \frac{1}{56}$
   b) Hence, calculate $P(3 \leq X < 8)$.
   c) Determine the values of $E(X)$ and $Var(X)$. Thus, evaluate $Var(\sqrt{3}\,X - 1)$

10. The number of batteries sold at a service center on any particular day follows a Poisson distribution with mean λ.
    a) If the probability of selling exactly 4 batteries divided by the probability of selling exactly 2 batteries is $\frac{225}{12}$, show that $\lambda = 15$.
    b) On any particular day, calculate the probability that the service center sells between 5 and 14 batteries.
    c) Given that the probability of selling less than k batteries on any particular day is 0.917, find the value of k.
    d) Find the probability that exactly 40 batteries are sold in 2 working days. Give your answer in four decimal places.

END OF QUESTION PAPER

1. a) Evaluate $\int x(1-x)^8\,dx$ by using a suitable substitution.

   b) Find the area of the region bounded by the curve $y = x\cos x$ and $x - axis$ between $x = 0$ and $x = \frac{\pi}{2}$. Give your solution in term of π.

SOLUTION

(a) $\int x(1-x)^8\,dx$

$u = 1 - x \quad \rightarrow \quad x = 1 - u$

$\frac{du}{dx} = -1$

$dx = -du$

$$\int x(1-x)^8\,dx = \int (1-u)u^8(-du)$$

$$= -\int (1-u)u^8 du$$

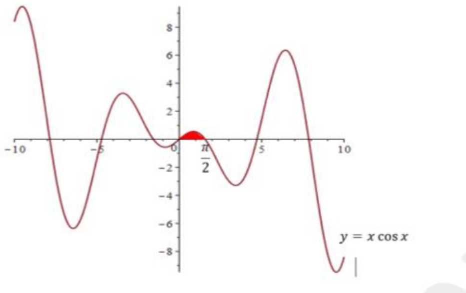
$$= -\int u^8 - u^9 du$$

$$= -\left[\frac{u^9}{9} - \frac{u^{10}}{10}\right] + C$$

$$= -\left[\frac{(1-x)^9}{9} - \frac{(1-x)^{10}}{10}\right] + C$$

$$= \frac{(1-x)^{10}}{10} - \frac{(1-x)^9}{9} + C$$

(b)



$$Area = \int_0^{\frac{\pi}{2}} x\cos x \; dx$$

$$\int x\cos x \; dx$$

$$u = x \qquad dv = \cos x \; dx$$

$$\frac{du}{dx} = 1 \qquad \int dv = \int \cos x \; dx$$

$$du = dx \qquad v = \sin x$$

$$\int u \; dv = uv - \int v \; du$$

$$Area = (x)(\sin x) - \int (\sin x)\,(dx)$$

$$= x\sin x - (-\cos x)$$

$$= x\sin x + \cos x$$

$$Area = \int_0^{\frac{\pi}{2}} x\cos x \; dx$$

$$= [x\sin x + \cos x]_0^{\frac{\pi}{2}}$$

$$= \left[\frac{\pi}{2}\sin\left(\frac{\pi}{2}\right) + \cos\left(\frac{\pi}{2}\right)\right] - [0\sin 0 + \cos(0)]$$

$$= \left[\frac{\pi}{2} + 0\right] - [0 + 1]$$

$$= \frac{\pi}{2} - 1 \;\; unit^2$$

2. Solve $\frac{dy}{dx} + 2(x+1)y^2 = 0$, given that $y = 1$ when $x = 1$. Express y in terms of x.

SOLUTION

$$\frac{dy}{dx} + 2(x+1)y^2 = 0$$

$$\frac{dy}{dx} = -2(x+1)y^2$$

$$y^{-2}dy = -2(x+1)dx$$

$$\int y^{-2}dy = -2\int (x+1)dx$$

$$\frac{y^{-1}}{-1} = -2\left(\frac{x^2}{2} + x\right) + C$$

$$-\frac{1}{y} = -x^2 - 2x + C$$

$$When\ x = 1, y = 1$$

$$-\frac{1}{1} = -1^2 - 2(1) + C$$

$$-1 = -1 - 2 + C$$

$$C = 2$$

$$-\frac{1}{y} = -x^2 - 2x + 2$$

$$-1 = (-x^2 - 2x + 2)y$$

$$y = \frac{1}{x^2 + 2x - 2}$$

3. Use Newton-Raphson method to solve the equation $e^x - x - 2 = 0$ correct to four decimal places by taking $x_1 = 1$ as the first approximation.

SOLUTION

$e^x - x - 2 = 0$

$f(x) = e^x - x - 2$

$f'(x) = e^x - 1$

$$x_{n+1} = x_n - \frac{f(x)}{f'(x)}$$

$$x_{n+1} = x_n - \frac{e^x - x - 2}{e^x - 1}$$

$x_1 = 1$

$$x_2 = 1 - \frac{e^1 - 1 - 2}{e^1 - 1} = 1.16395$$

$$x_3 = 1.16395 - \frac{e^{1.16395} - 1.16395 - 2}{e^{1.163951} - 1} = 1.14642$$

$$x_4 = 1.14642 - \frac{e^{1.14642} - 1.14642 - 2}{e^{1.14642} - 1} = 1.14619$$

$$x_4 = 1.14619 - \frac{e^{1.14619} - 1.14619 - 2}{e^{1.14619} - 1} = 1.14619$$

$\therefore x = 1.1462$

4. An ellipse $Ax^2 + y^2 + Bx + Cy + 1 = 0$ passes through points $(0,1), (1,-1)\ and\ (2,1)$.
   a) Find the equation of the ellipse in the standard form. Hence, state the centre and vertices of the ellipse.
   b) Find the foci of th ellipse.
   c) Sketch the graph of the ellipse.

SOLUTION

$Ax^2 + y^2 + Bx + Cy + 1 = 0$

$At\ (0,1)$

$A(0^2) + 1^2 + B0 + C(1) + 1 = 0$

$C = -2$

$At\ (1,-1)\ and\ C = -2$

$A(1^2) + (-1)^2 + B(1) + (-2)(-1) + 1 = 0$

$A + 1 + B + 2 + 1 = 0$

$A + B = -4$ .... (1)

$At\ (2,1)\ and\ C = -2$

$A(2)^2 + (1)^2 + B(2) + (-2)(1) + 1 = 0$

$4A + 1 + 2B - 2 + 1 = 0$

$4A + 2B = 0$ .... (2)

$(1)X2$

$2A + 2B = -8$ .... (3)

$(2) - (3)$

$2A = 8$

$A = 4$

$B = -8$

The equation of ellipse:

$4x^2 + y^2 - 8x - 2y + 1 = 0$

$4x^2 - 8x + y^2 - 2y = -1$

$4(x^2 - 2x) + (y^2 - 2y) = -1$

$4(x^2 - 2x + 1^2 - 1^2) + (y^2 - 2y + 1^2 - 1^2) = -1$

$$4[(x-1)^2-1]+[(y-1)^2-1]=-1$$
$$4(x-1)^2-4+(y-1)^2-1=-1$$
$$4(x-1)^2+(y-1)^2=4$$
$$\frac{4(x-1)^2}{4}+\frac{(y-1)^2}{4}=\frac{4}{4}$$
$$\frac{(x-1)^2}{1^2}+\frac{(y-1)^2}{2^2}=1$$

Standard Equation of Ellipse

$$\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$$

$a=1, b=2, h=1, k=1$

$Centre: C(h,k)=C(1,1)$

$Vertices: V(h,k\pm b) \Rightarrow V_1(1,1+2), V_2(1,1-2)$

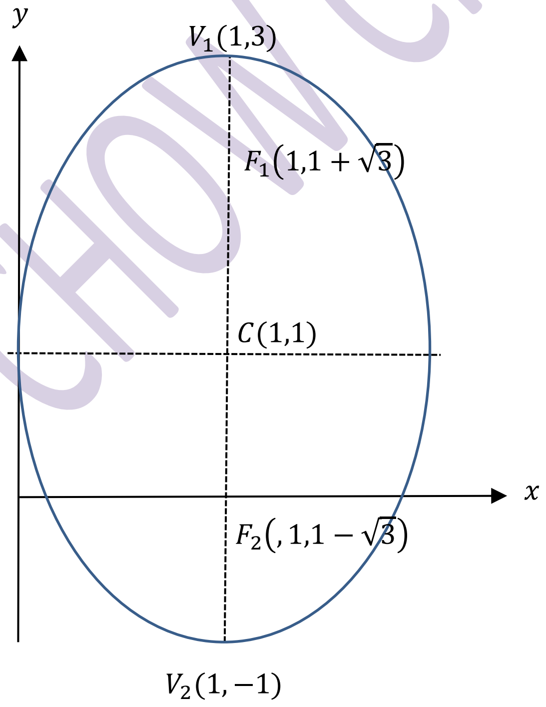
$V_1(1,3), V_2(1,-1)$

(4b)

$c=\sqrt{2^2-1^2}=\sqrt{3}$

$Foci: F(h,k\pm c)=F\left(1,1\pm\sqrt{3}\right)$

(4c)



5. The line L_1 and L_2 passes through the point $R(2,4,-3)$ and $S(8,-5,9)$in the direction of $2\boldsymbol{i}-3\boldsymbol{j}+4\boldsymbol{k}$ *and* $\boldsymbol{i}-2\boldsymbol{j}+3\boldsymbol{k}$, respectively.
   a) State the equations for lines L_1 and L_2 in the vector form. Hence, calculate the acute angle between the lines L_1 and L_2.
   b) Find the equation of plane containing the line L_1 and the point $(7,-3,5)$ in the Cartesian form.
   c) Determine whether the line L_2 is parallel to the plane $x+5y+3z=5$.

SOLUTION

(5a)

L_1: $\boldsymbol{a_1}=2\boldsymbol{i}+4\boldsymbol{j}-3\boldsymbol{k}$; $\quad\boldsymbol{v_1}=2\boldsymbol{i}-3\boldsymbol{j}+4\boldsymbol{k}$

L_2: $\boldsymbol{a_2}=8\boldsymbol{i}-5\boldsymbol{j}+9\boldsymbol{k}$; $\quad\boldsymbol{v_2}=\boldsymbol{i}-2\boldsymbol{j}+3\boldsymbol{k}$

$\boldsymbol{r}=\boldsymbol{a}+t\boldsymbol{v}$

Equation of $\boldsymbol{L_1}$ ***and*** $\boldsymbol{L_2}$

L_1: $\boldsymbol{r_1}=(2\boldsymbol{i}+4\boldsymbol{j}-3\boldsymbol{k})+t_1(2\boldsymbol{i}-3\boldsymbol{j}+4\boldsymbol{k})$

L_2: $\boldsymbol{r_2}=(8\boldsymbol{i}-5\boldsymbol{j}+9\boldsymbol{k})+t_2(\boldsymbol{i}-2\boldsymbol{j}+3\boldsymbol{k})$

Angle between $\boldsymbol{L_1}$ ***and*** $\boldsymbol{L_2}(\boldsymbol{\theta})$

$$Cos\ \theta=\frac{\boldsymbol{v_1}.\ \boldsymbol{v_2}}{|\boldsymbol{v_1}||\boldsymbol{v_2}|}$$

$$Cos\ \theta=\frac{(2i-3j+4k).(i-2j+3k)}{|2i-3j+4k||i-2j+3k|}$$

$$=\frac{(2)(1)+(-3)(-2)+(4)(3)}{\sqrt{(2)^2+(-3)^2+(4)^2}\sqrt{(1)^2+(-2)^2+(3)^2}}$$

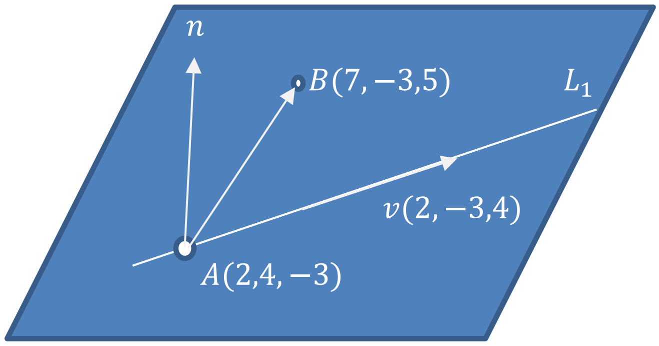
$$=\frac{20}{\sqrt{29}\sqrt{14}}$$

$$=0.9926$$

$$\theta=cos^{-1}(0.9926)$$

$$=6.98°$$

(5b)



$\boldsymbol{Equation\ of\ plane}$

$\boldsymbol{r.n = a.n}$

$\boldsymbol{a} = 7\boldsymbol{i} - 3\boldsymbol{j} + 5\boldsymbol{k}$

$\boldsymbol{n} = \boldsymbol{v}\ \boldsymbol{x}\ \overrightarrow{\boldsymbol{AB}}$

$\boldsymbol{v} = 2\boldsymbol{i} - 3\boldsymbol{j} + 4\boldsymbol{k}$

$\overrightarrow{\boldsymbol{AB}} = \overrightarrow{\boldsymbol{OB}} - \overrightarrow{\boldsymbol{OA}}$

$= (7\boldsymbol{i} - 3\boldsymbol{j} + 5\boldsymbol{k}) - (2\boldsymbol{i} + 4\boldsymbol{j} - 3\boldsymbol{k})$

$= (\boldsymbol{5i - 7j} + 8\boldsymbol{k})$

$$\boldsymbol{n} = \begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ \boldsymbol{2} & \boldsymbol{-3} & \boldsymbol{4} \\ \boldsymbol{5} & \boldsymbol{-7} & \boldsymbol{8} \end{vmatrix}$$

$= (-24 + 28)\boldsymbol{i} - (16 - 20)\boldsymbol{j} + (-14 + 15)\boldsymbol{k}$

$= 4\boldsymbol{i} + 4\boldsymbol{j} + \boldsymbol{k}$

$\boldsymbol{Equation\ of\ plane}$

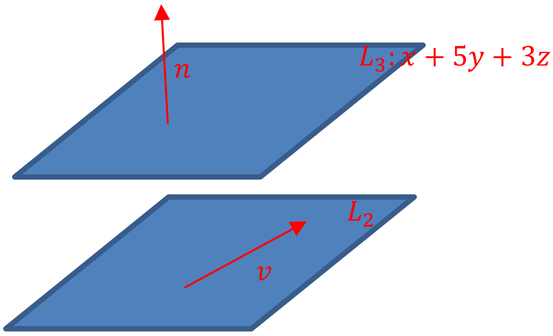
$\boldsymbol{r.n = a.n}$

$(x\boldsymbol{i} + y\boldsymbol{j} + z\boldsymbol{k}).(4\boldsymbol{i} + 4\boldsymbol{j} + \boldsymbol{k}) = (7\boldsymbol{i} - 3\boldsymbol{j} + 5\boldsymbol{k}).(4\boldsymbol{i} + 4\boldsymbol{j} + \boldsymbol{k})$

$4x + 4y + z = 28 - 12 + 5$

$4x + 4y + z = 21$

(5c)



From the diagram, if $\boldsymbol{L_2}$ *is parallel to* $\boldsymbol{L_3}$*, then* $\boldsymbol{v}$ *must be perpendicular to* $\boldsymbol{n}$

$\boldsymbol{v} = \boldsymbol{i} - 2\boldsymbol{j} + 3\boldsymbol{k}$

$\boldsymbol{n} = \boldsymbol{i} + 5\boldsymbol{j} + 3\boldsymbol{k}$

$\boldsymbol{v}.\boldsymbol{n} = (\boldsymbol{i} - 2\boldsymbol{j} + 3\boldsymbol{k}).(\boldsymbol{i} + 5\boldsymbol{j} + 3\boldsymbol{k})$

$= \mathbf{1 - 10 + 9}$

$= \mathbf{0}$

Since $\boldsymbol{v.n = 0}$***, there for line*** $\boldsymbol{L_1}$ ***is parallel to the plane*** $\boldsymbol{x + 5y + 3z = 5}$.

6. **TABLE 1** shows the frequency distribution of the diameter of 100 pebbles which are measured to the nearest milliType equation here.meter(mm).

TABLE 1

Diameter (mm)	Frequency
10 – 14	22
15 – 19	20
20 – 24	25
25 – 29	15
30 - 34	18

Calculate the

a) mean.

b) mode.

c) median.

SOLUTION

Diameter (mm)	Class Boundary	Mid Point (x)	Frequency (f)	Cummulative Frequency (F)
10 – 14	9.5 – 14.5	12	22	22
15 – 19	14.5 – 19.5	17	20	42
20 – 24	19.5 – 24.5	22	25	67
25 – 29	24.5 – 29.5	27	15	82
30 - 34	30.5 – 34.5	32	18	100
		$\sum f$	100	

(6a)

$$Mean = \frac{\sum fx}{\sum f}$$

$$Mean = \frac{(12)(22) + (17)(20) + (22)(25) + (27)(15) + (32)(18)}{100}$$

$$= \frac{2135}{100}$$

$$= 21.35$$

(6b)

$$Mode = L_m + \left[\frac{d_1}{d_1 + d_2}\right] C$$

$$Mode = 19.5 + \left[\frac{5}{5 + 10}\right] 5$$

$$= 21.17$$

(6c)

$$Median = L_m + \left[\frac{\frac{n}{2} - F_{k-1}}{f_k}\right] C$$

$$Median = 19.5 + \left[\frac{\frac{100}{2} - 42}{25}\right] 5$$

$$= 21.1$$

7. A committee of 5 is to be selected from a group of 7 men and 6 women. How many different committees could be formed if
   a) there is no woman in the committee?
   b) a particular man must be in the committee and the remaining has equal number of men and women?
   c) at least 3 men are in the committee?

SOLUTION

(7a)

$if\ there\ is\ no\ woman\ in\ the\ committee.$

$^7C_5\ \ ^6C_0$ =21

(7b)

$^1C_1\ x\ ^6C_2\ x\ ^6C_2\ \ = 225$

(7c)

$At\ least\ 3\ men\ in\ the\ committee.$

Men	Women	Total
3	2	$^7C_3\ \ ^6C_2\ \ = 525$
4	1	$^7C_4\ \ ^6C_1$ =210
5	0	$^7C_5\ \ ^6C_0$ =21
	Total	756

The number of different committee could be formed = 756

8. The probabilities of events X and Y are given as $P(X)=\frac{3}{5}$, $P(X'|Y)=\frac{31}{45}$ and $P(X\cap Y)=\frac{2}{25}$.

a) Show that $P(Y)=\frac{9}{35}$

b) Find $P(X\cup Y')$.

SOLUTION

(8a)

$$P(X)=\frac{3}{5} \qquad P(X\cap Y)=\frac{2}{25}$$

$$P(X'|Y)=\frac{31}{45}$$

$$\frac{P(X'\cap Y)}{P(Y)}=\frac{31}{45}$$

$$\frac{P(Y)-P(X\cap Y)}{P(Y)}=\frac{31}{45}$$

$$\frac{P(Y)-\frac{2}{25}}{P(Y)}=\frac{31}{45}$$

$$1-\frac{\frac{2}{25}}{P(Y)}=\frac{31}{45}$$

$$\frac{\frac{2}{25}}{P(Y)}=1-\frac{31}{45}$$

$$\frac{\frac{2}{25}}{P(Y)}=\frac{14}{45}$$

$$P(Y)=\frac{2}{25}x\frac{45}{14}$$

$$P(Y)=\frac{9}{35}$$

(8b)

$$P(X \cup Y') = P(X) + P(Y') - P(X \cap Y')$$

$$P(Y') = 1 - P(Y) = 1 - \frac{9}{35} = \frac{26}{35}$$

$$P(X \cap Y') = P(X) - P(X \cap Y) = \frac{3}{5} - \frac{2}{25} = \frac{13}{25}$$

$$P(X \cup Y') = \frac{3}{5} + \frac{26}{35} - \frac{13}{25}$$

$$= \frac{144}{175}$$

9. A discrete random variable X has the probability distribution function

$$f(x) = \begin{cases} \frac{x+1}{16}, & x = 2,3,4 \\ kx, & x = 6,8 \\ 0, & otherwise \end{cases}$$

a) Show that $k = \frac{1}{56}$

b) Hence, calculate $P(3 \le X < 8)$.

c) Determine the values of $E(X)$ and $Var(X)$. Thus, evaluate $Var(\sqrt{3}\,X - 1)$

SOLUTION

$$f(x) = \begin{cases} \frac{x+1}{16}, & x = 2,3,4 \\ kx, & x = 6,8 \\ 0, & otherwise \end{cases}$$

x	2	3	4	6	8
$P(X = x)$	$\frac{3}{16}$	$\frac{4}{16}$	$\frac{5}{16}$	$6k$	$8k$

(9a)

$$\sum P(X = x) = 1$$

$$\frac{3}{16} + \frac{4}{16} + \frac{5}{16} + 6k + 8k = 1$$

$$\frac{12}{16} + 14k = 1$$

$$14k = \frac{1}{4}$$

$$k = \frac{1}{56}$$

(9b)

x	2	3	4	6	8
$P(X=x)$	$\frac{3}{16}$	$\frac{4}{16}$	$\frac{5}{16}$	$\frac{3}{28}$	$\frac{1}{7}$

$$P(3 \leq X < 8) = P(X=3) + P(X=4) + P(X=6)$$
$$= \frac{4}{16} + \frac{5}{16} + \frac{3}{28}$$
$$= \frac{75}{112}$$

(9c)

$$E(X) = \sum x\, P(X=x)$$

$$E(X) = (2)\left(\frac{3}{16}\right) + (3)\left(\frac{4}{16}\right) + (4)\left(\frac{5}{16}\right) + (6)\left(\frac{3}{28}\right) + (8)\left(\frac{1}{7}\right)$$
$$= \frac{233}{56}$$

$$E(X^2) = \sum x^2\, P(X=x)$$

$$E(X^2) = (2)^2\left(\frac{3}{16}\right) + (3)^2\left(\frac{4}{16}\right) + (4)^2\left(\frac{5}{16}\right) + (6)^2\left(\frac{3}{28}\right) + (8)^2\left(\frac{1}{7}\right)$$
$$= 21$$

$$Var(X) = E(X^2) - [E(X)]^2$$
$$= 21 - \left(\frac{233}{56}\right)^2$$
$$= 3.688$$

$$Var(ax+b) = a^2 Var(x)$$
$$Var(\sqrt{3}x - 1) = \sqrt{3}^2\, x\, 3.688$$
$$= 11.065$$

10. The number of batteries sold at a service center on any particular day follows a Poisson distribution with mean λ.

a) If the probability of selling exactly 4 batteries divided by the probability of selling exactly 2 batteries is $\frac{225}{12}$, show that $\lambda = 15$.

b) On any particular day, calculate the probability that the service center sells between 5 and 14 batteries.

c) Given that the probability of selling less than k batteries on any particular day is 0.917, find the value of k.

d) Find the probability that exactly 40 batteries are sold in 2 working days. Give your answer in four decimal places.

SOLUTION

(10a)

For Poisson Distribution

$$P(X = x) = \frac{e^{-\lambda}\lambda^x}{x!}$$

$$\frac{P(X = 4)}{P(X = 2)} = \frac{225}{12}$$

$$\frac{\frac{e^{-\lambda}\lambda^4}{4!}}{\frac{e^{-\lambda}\lambda^2}{2!}} = \frac{225}{12}$$

$$\left(\frac{e^{-\lambda}\lambda^4}{4!}\right)\left(\frac{2!}{e^{-\lambda}\lambda^2}\right) = \frac{225}{12}$$

$$\frac{\lambda^2}{12} = \frac{225}{12}$$

$$\lambda^2 = 225$$

$$\lambda = 15$$

(10b)

$$X \sim Po(15)$$

$$P(5 < X < 14) = P(X \geq 6) - P(X \geq 14)$$

$$= 0.9975 - 0.6368$$

$$= 0.3604$$

(10c)

$P(X < k) = 0.917$

$1 - P(X \geq k) = 0.917$

$P(X \geq k) = 0.0830$

From statistical table:

$k = 21$

(10d)

$X \sim Po(30)$

$$P(X = 40) = \frac{e^{-30}(30)^{40}}{40!}$$

$$= 0.0139$$