KEMENTERIAN PENDIDIKAN MALAYSIA

QS 015/1

Matriculation Programme Examination

Semester I

Session 2014/2015

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1. Solve the equation $3^x + 3^{(3-x)} = 12$.

2. Solve the inequality $\frac{1}{6-x} < \frac{1}{x-1}$.

3. Given matrices $A = \begin{bmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ a & b & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 & 0 \\ z & 1 & 0 \\ x & y & 1 \end{bmatrix}$ where B is the inverse of A. Find x, y and z in terms of a and b.

4. Using algebraic method, find the least value of n for which the sum of the first n terms of a geometric series

$$0.88 + (0.88)^2 + (0.88)^3 + (0.88)^4 + \cdots$$

is greater than half of its sum to infinity.

5. (a) State the interval for x such that the expansion for $(4+3x)^{\frac{3}{2}}$ is valid.

(b) Expand $(4+3x)^{\frac{3}{2}}$ in ascending power of x up to the term in x^3.

(c) Hence, by substituting an appropriate value of x, evaluate $(5)^{\frac{3}{2}}$ correct to three decimal places.

6. (a) Given $f(x) = 2x + 1$ and $g(x) = x^2 + 2x - 1$.

(i) Find $(f - g)(x)$.

(ii) Evaluate $(3g - 2)(1)$.

(b) Given $f(x) = \sqrt{2x + \frac{1}{2}}$. State the domain and range of $f(x)$. Hence, on the same axes, sketch the graph of $f(x)$ and $f^{-1}(x)$.

7. Let $z = a + bi$ be a nonzero complex number.

(a) Show that $\frac{1}{z} = \frac{\bar{z}}{|z|^2}$.

(b) Show that if $\bar{z} = -z$, then z is a complex number with only an imaginary part.

(c) Find the value of a and b if $z(2-i) = (\bar{z}+1)(1+i)$.

8. (a) Solve the for the following equation $|6x^2 + x - 11| = 4$.

(b) Find the solution set for the inequality

$$2 - \left(\frac{x+2}{x-4}\right) < 5$$

9. Two companies P and Q decided to award prizes to their employees for three work ethical values, namely punctuality (x), creativity (y) and efficiency (x). Company P decided to award a total of RM3850 for the three values to 6, 2 and 3 employees respectively, while company Q decided to award RM3200 for the three values to 4, 1 and 5 employees respectively. The total amount for all the three prizes is RM1000.

(a) Construct a system of linear equations to represent the above situation.

(b) By forming a matrix equation, solve this equation system using the elimination method.

(c) With the same total amount of money spent by company P and Q, is it possible for company P to award 15 employees for their creativity instead of 2 employees? Give your reason.

10. (a) Determine wheather $f(x) = \frac{1}{x-4}$ and $g(x) = \frac{4x+1}{x}$ are inverse function of each other by computing their composite functions.

(b) Given $f(x) = \ln(1 - 3x)$.

(i) Determine the domain and range of $f(x)$. Then sketch the graph of $f(x)$.

(ii) Find $f^{-1}(x)$, if it exists. Hence, state the domain and range of $f^{-1}(x)$.

END OF QUESTION PAPER

1. Solve the equation $3^x + 3^{(3-x)} = 12$.

SOLUTION

$$3^x + 3^{(3-x)} = 12$$
$$3^x + 3^3.3^{-x} = 12$$
$$3^x + 3^3.\frac{1}{3^x} = 12$$
$$3^x + \frac{27}{3^x} = 12$$

Let $y = 3^x$

$$y + \frac{27}{y} = 12$$
$$\frac{y^2 + 27}{y} = 12$$
$$y^2 + 27 = 12y$$
$$y^2 - 12y + 27 = 0$$
$$(y-9)(y-3) = 0$$

$(y-9) = 0$	$(y-3) = 0$
$y = 9$	$y = 3$
$3^x = 9$	$3^x = 3$
$x = 2$	$x = 1$

2. Solve the inequality $\frac{1}{6-x} < \frac{1}{x-1}$.

SOLUTION

$$\frac{1}{6-x} < \frac{1}{x-1}$$

$$\frac{1}{6-x} - \frac{1}{x-1} < 0$$

$$\frac{(x-1)-(6-x)}{(6-x)(x-1)} < 0$$

$$\frac{x-1-6+x}{(6-x)(x-1)} < 0$$

$$\frac{2x-7}{(6-x)(x-1)} < 0$$

$2x - 7 = 0$ $6 - x = 0$ $x - 1 = 0$

$x = \frac{7}{2}$ $x = 6$ $x = 1$

	$(-\infty, 1)$	$\left(1, \frac{7}{2}\right)$	$\left(\frac{7}{2}, 6\right)$	$(6, \infty)$
$2x-7$	-	-	+	+
$6-x$	+	+	+	-
$x-1$	-	+	+	+
$\frac{2x-7}{(6-x)(x-1)}$	+	-	+	-

$$\therefore \left(1, \frac{7}{2}\right) \cup (6, \infty)$$

3. Given matrices $A = \begin{bmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ a & b & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 & 0 \\ z & 1 & 0 \\ x & y & 1 \end{bmatrix}$ where B is the inverse of A. Find x, y and z in terms of a and b.

SOLUTION

$A = \begin{bmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ a & b & 1 \end{bmatrix}$ $\qquad$ $B = \begin{bmatrix} 1 & 0 & 0 \\ z & 1 & 0 \\ x & y & 1 \end{bmatrix}$

$Since\ B$ is the inverse of A ➔ $AB = I$

$$AB = I$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ a & b & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ z & 1 & 0 \\ x & y & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1+0+0 & 0+0+0 & 0+0+0 \\ 4+z+0 & 0+1+0 & 0+0+0 \\ a+bz+x & 0+b+y & 0+0+1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 4+z & 1 & 0 \\ a+bz+x & b+y & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$4 + z = 0$ ➔ $z = -4$

$a + bz + x = 0$ ➔ $x = -a + 4b$

$b + y = 0$ ➔ $y = -b$

4. Using algebraic method, find the least value of n for which the sum of the first n terms of a geometric series

$$0.88 + (0.88)^2 + (0.88)^3 + (0.88)^4 + \cdots$$

is greater than half of its sum to infinity.

SOLUTION

Geometric Series

$$s_n = \frac{a(1-r^n)}{1-r}$$

$$s_{\infty} = \left(\frac{a}{1-r}\right)$$

$$s_n > \frac{1}{2}s_\infty$$

$$a = 0.88, r = 0.88$$

$$\frac{a(1-r^n)}{1-r} > \frac{1}{2}\left(\frac{a}{1-r}\right)$$

$$\frac{(0.88)[1-(0.88)^n]}{1-0.88} > \frac{1}{2}\left(\frac{0.88}{1-0.88}\right)$$

$$[1-(0.88)^n] > \frac{1}{2}\left(\frac{0.88}{1-0.88}\right)\left(\frac{1-0.88}{0.88}\right)$$

$$1-(0.88)^n > \frac{1}{2}$$

$$1-\frac{1}{2} > (0.88)^n$$

$$\frac{1}{2} > (0.88)^n$$

$$(0.88)^n < \frac{1}{2}$$

$$log\,(0.88)^n < log\left(\frac{1}{2}\right)$$

$$n\log 0.88 < -0.3010$$

$$n(-0.0555) < -0.3010$$

$$-0.0555n < -0.3010$$

$$0.3010 < 0.0555n$$

$$0.0555n > 0.3010$$

$$n > \frac{0.3010}{0.0555}$$

$$n > 5.422$$

$$n = 6$$

5. (a) State the interval for x such that the expansion for $(4+3x)^{\frac{3}{2}}$ is valid.

(b) Expand $(4+3x)^{\frac{3}{2}}$ in ascending power of x up to the term in x^3.

(c) Hence, by substituting an appropriate value of x, evaluate $(5)^{\frac{3}{2}}$ correct to three decimal places.

SOLUTION

a) $(4+3x)^{\frac{3}{2}} = \left[4\left(1+\frac{3}{4}x\right)\right]^{\frac{3}{2}}$

$$\textit{The expansion is valid for}$$
$$-1 < \frac{3}{4}x < 1$$
$$-\frac{4}{3} < x < \frac{4}{3}$$

b) $(4+3x)^{\frac{3}{2}} = \left[4\left(1+\frac{3}{4}x\right)\right]^{\frac{3}{2}}$

$$= 4^{\frac{3}{2}}\left(1+\frac{3}{4}x\right)^{\frac{3}{2}}$$
$$= 8\left(1+\frac{3}{4}x\right)^{\frac{3}{2}}$$
$$= 8\left[1+\frac{\left(\frac{3}{2}\right)}{1!}\left(\frac{3}{4}x\right)+\frac{\left(\frac{3}{2}\right)\left(\frac{1}{2}\right)}{2!}\left(\frac{3}{4}x\right)^2+\frac{\left(\frac{3}{2}\right)\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)}{3!}\left(\frac{3}{4}x\right)^3+\cdots\right]$$
$$= 8\left[1+\frac{9}{8}x+\frac{3}{8}\left(\frac{9}{16}x^2\right)-\frac{3}{48}\left(\frac{27}{64}x^3\right)+\cdots\right]$$
$$= 8\left[1+\frac{9}{8}x+\frac{27}{128}x^2-\frac{81}{3072}x^3+\cdots\right]$$
$$= 8\left[1+\frac{9}{8}x+\frac{27}{128}x^2-\frac{27}{1024}x^3+\cdots\right]$$
$$= 8+9x+\frac{27}{16}x^2-\frac{27}{128}x^3+\cdots$$

c) $(4+3x)^{\frac{3}{2}} = 8+9x+\frac{27}{16}x^2-\frac{27}{128}x^3+\cdots$

$$(4+3x)^{\frac{3}{2}} = (5)^{\frac{3}{2}}$$
$$4+3x = 5$$

$$3x = 1$$

$$x = \frac{1}{3}$$

$$(4+3x)^{\frac{3}{2}} = 8 + 9x + \frac{27}{16}x^2 - \frac{27}{128}x^3 + \cdots$$

$$\left[4 + 3\left(\frac{1}{3}\right)\right]^{\frac{3}{2}} = 8 + 9\left(\frac{1}{3}\right) + \frac{27}{16}\left(\frac{1}{3}\right)^2 - \frac{27}{128}\left(\frac{1}{3}\right)^3 + \cdots$$

$$[5]^{\frac{3}{2}} = 8 + 9\left(\frac{1}{3}\right) + \frac{27}{16}\left(\frac{1}{3}\right)^2 - \frac{27}{128}\left(\frac{1}{3}\right)^3 + \cdots$$

$$= 11.180$$

6. (a) Given $f(x) = 2x + 1$ and $g(x) = x^2 + 2x - 1$.

(i) Find $(f - g)(x)$.

(ii) Evaluate $(3g - 2f)(1)$.

(b) Given $f(x) = \sqrt{2x + \frac{1}{2}}$. State the domain and range of $f(x)$. Hence, on the same axes, sketch the graph of $f(x)$ and $f^{-1}(x)$.

SOLUTION

a) $f(x) = 2x + 1$ and $g(x) = x^2 + 2x - 1$

i. $(f - g)(x) = f(x) - g(x)$

$$= (2x + 1) - (x^2 + 2x - 1)$$
$$= 2x + 1 - x^2 - 2x + 1$$
$$= -x^2 + 2$$

ii. $(3g - 2f)(x) = 3g(x) - 2f(x)$

$$= 3(x^2 + 2x - 1) - 2(2x + 1)$$
$$= 3x^2 + 6x - 3 - 4x - 2$$
$$= 3x^2 + 2x - 5$$

$$(3g - 2f)(1) = 3(1)^2 + 2(1) - 5$$
$$= 3 + 2 - 5$$
$$= 0$$

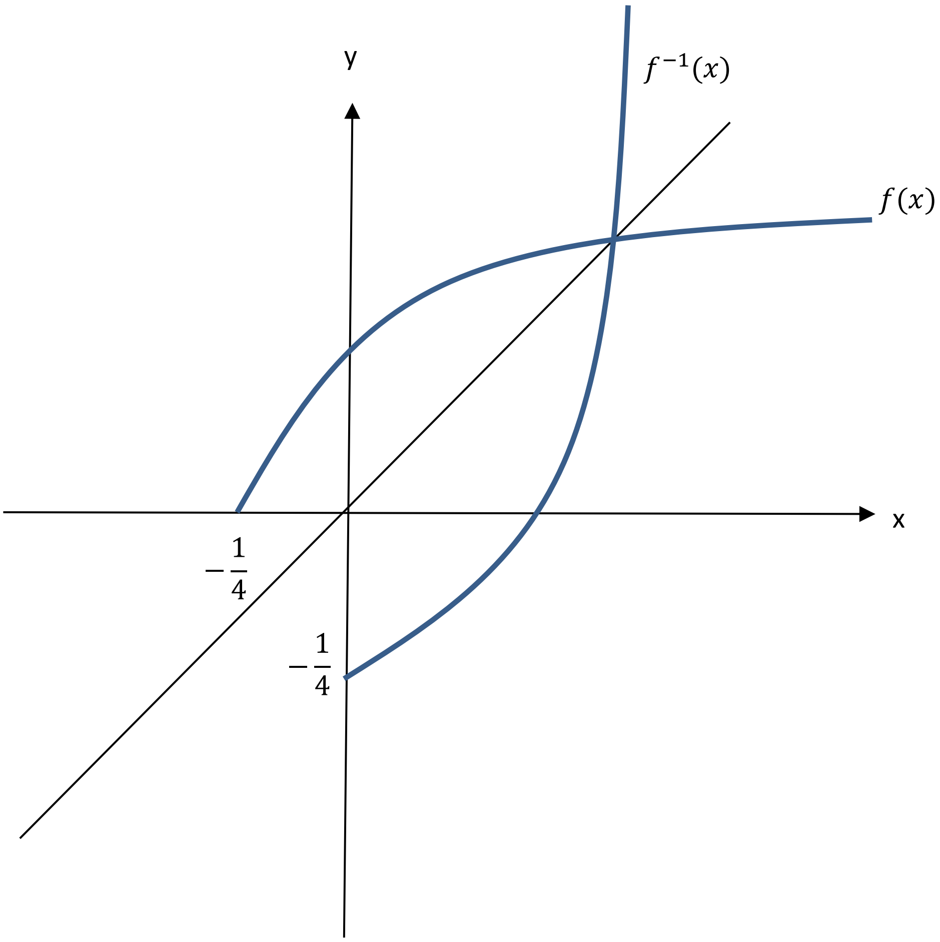
b) $f(x) = \sqrt{2x + \frac{1}{2}}$

D_f: $2x + \frac{1}{2} \geq 0$

$$2x \geq -\frac{1}{2}$$
$$x \geq -\frac{1}{4}$$

$$D_f = \left\{x : x \geq -\frac{1}{4}\right\}$$

$$R_f = \{y : y \geq 0\}$$


y
$f^{-1}(x)$
$f(x)$
x
$-\frac{1}{4}$
$-\frac{1}{4}$

7. Let $z = a + bi$ be a nonzero complex number.

   a. Show that $\frac{1}{z} = \frac{\bar{z}}{|z|^2}$.

   b. Show that if $\bar{z} = -z$, then z is a complex number with only an imaginary part.

   c. Find the value of a and b if $z(2 - i) = (\bar{z} + 1)(1 + i)$.

SOLUTION

a) $z = a + bi$

$$\frac{\bar{z}}{|z|^2} = \frac{a-bi}{|a+bi|^2}$$

$$= \frac{a - bi}{\left(\sqrt{a^2 + b^2}\right)^2}$$

$$= \frac{a - bi}{a^2 + b^2}$$

$$= \frac{a - bi}{a^2 + b^2}\left(\frac{a + bi}{a + bi}\right)$$

$$= \frac{a^2 + abi - abi - (bi)^2}{(a^2 + b^2)(a + bi)}$$

$$= \frac{a^2 - b^2 i^2}{(a^2 + b^2)(a + bi)}$$

$$= \frac{a^2 - b^2(-1)}{(a^2 + b^2)(a + bi)}$$

$$= \frac{a^2 + b^2}{(a^2 + b^2)(a + bi)}$$

$$= \frac{1}{(a + bi)}$$

$$= \frac{1}{z}$$

b) $\overline{z} = -z$

$a - bi = -(a + bi)$

$a - bi = -a - bi$

$a = -a$

$2a = 0$

$a = 0$

$Since\ z = a + bi$

$= 0 + bi$

$= bi$

$\therefore$ z is a complex number with only an imaginary part when $\overline{z} = -z$

c) $z(2 - i) = (\overline{z} + 1)(1 + i)$

$(a + bi)(2 - i) = [(a - bi) + 1](1 + i)$

$2a + 2bi - ai - bi^2 = (a - bi)(1 + i) + (1 + i)$

$2a + 2bi - ai + b = (a - bi + ai - bi^2) + (1 + i)$

$2a + b + (2b - a)i = a - bi + ai + b + 1 + i$

$2a + (2b - a)i = (a + 1) + (a - b + 1)i$

By equating real part

$2a = a + 1$

$a = 1$

By equating imaginary part

$(2b - a) = (a - b + 1)$

$3b - 2a = 1$

$3b - 2(1) = 1$

$3b - 2 = 1$

$3b = 3$

$b = 1$

8. (a) Solve the for the following equation $|6x^2 + x - 11| = 4$.

(b) Find the solution set for the inequality

$$2 - \left(\frac{x+2}{x-4}\right) < 5$$

$|x| = a \Leftrightarrow x = a \text{ or } x = -a$

SOLUTION

a) $|6x^2 + x - 11| = 4$

$6x^2 + x - 11 = 4$ or $6x^2 + x - 11 = -4$

$6x^2 + x - 15 = 0$ $\quad$ $6x^2 + x - 7 = 0$

$(3x + 5)(2x - 3) = 0$ $\quad$ $(6x + 7)(x - 1) = 0$

$3x + 5 = 0 \quad 2x - 3 = 0$ $\quad$ $6x + 7 = 0 \quad x - 1 = 0$

$x = -\frac{5}{3},\ x = \frac{3}{2}$ $\quad$ $x = -\frac{7}{6},\ x = 1$

$$\therefore = -\frac{5}{3}, \quad x = \frac{3}{2}, \quad x = -\frac{7}{6}, \quad x = 1$$

b) $2 - \left(\frac{x+2}{x-4}\right) < 5$

$$2 - \left(\frac{x+2}{x-4}\right) - 5 < 0$$

$$\frac{2(x-4) - (x+2) - 5(x-4)}{x-4} < 0$$

$$\frac{2x - 8 - x - 2 - 5x + 20}{x-4} < 0$$

$$\frac{-4x + 10}{x-4} < 0$$

$-4x + 10 = 0 \qquad x - 4 = 0$

$x = \frac{10}{4} = \frac{5}{2} \qquad x = 4$

	$\left(-\infty, \frac{5}{2}\right)$	$\left(\frac{5}{2}, 4\right)$	$(4, \infty)$
$-4x + 10$	+	-	-
$x - 4$	-	-	+
$\frac{-4x+10}{x-4}$	-	+	-

$$The\ solution\ set{:}\ \left\{x: x < \frac{5}{2} \cup x > 4\right\}$$

9. Two companies P and Q decided to award prizes to their employees for three work ethical values, namely punctuality (x), creativity (y) and efficiency (z). Company P decided to award a total of RM3850 for the three values to 6, 2 and 3 employees respectively, while company Q decided to award RM3200 for the three values to 4, 1 and 5 employees respectively. The total amount for all the three prizes is RM1000.

(a) Construct a system of linear equations to represent the above situation.

(b) By forming a matrix equation, solve this equation system using the elimination method.

(c) With the same total amount of money spent by company P and Q, is it possible for company P to award 15 employees for their creativity instead of 2 employees? Give your reason.

SOLUTION

a) $6x + 2y + 3z = 3850$

$4x + y + 5z = 3200$

$x + y + z = 1000$

b) $AX = B$

$$\begin{pmatrix} 6 & 2 & 3 \\ 4 & 1 & 5 \\ 1 & 1 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3850 \\ 3200 \\ 1000 \end{pmatrix}$$

$$\left(\begin{array}{ccc|c} 6 & 2 & 3 & 3850 \\ 4 & 1 & 5 & 3200 \\ 1 & 1 & 1 & 1000 \end{array}\right)$$

$R_3 \leftrightarrow R_3$ $$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 1000 \\ 4 & 1 & 5 & 3200 \\ 6 & 2 & 3 & 3850 \end{array}\right)$$

$R_2 = 4R_1 - R_2$ $$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 1000 \\ 0 & 3 & -1 & 800 \\ 6 & 2 & 3 & 3850 \end{array}\right)$$

$R_3 = 6R_1 - R_3$ $$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 1000 \\ 0 & 3 & -1 & 800 \\ 0 & 4 & 3 & 2150 \end{array}\right)$$

$R_3 = 4R_2 - 3R_3$ $$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 1000 \\ 0 & 3 & -1 & 800 \\ 0 & 0 & -13 & -3250 \end{array}\right)$$

$R_3 = \frac{1}{-13}R_3$ $\left(\begin{array}{ccc|c} 1 & 1 & 1 & 1000 \\ 0 & 3 & -1 & 800 \\ 0 & 0 & 1 & 250 \end{array}\right)$

$R_2 = R_2 + R_3$ $\left(\begin{array}{ccc|c} 1 & 1 & 1 & 1000 \\ 0 & 3 & 0 & 1050 \\ 0 & 0 & 1 & 250 \end{array}\right)$

$R_1 = R_1 - R_3$ $\left(\begin{array}{ccc|c} 1 & 1 & 0 & 750 \\ 0 & 3 & 0 & 1050 \\ 0 & 0 & 1 & 250 \end{array}\right)$

$R_2 = \frac{1}{3}R_2$ $\left(\begin{array}{ccc|c} 1 & 1 & 0 & 750 \\ 0 & 1 & 0 & 350 \\ 0 & 0 & 1 & 250 \end{array}\right)$

$R_1 = R_1 - R_2$ $\left(\begin{array}{ccc|c} 1 & 0 & 0 & 400 \\ 0 & 1 & 0 & 350 \\ 0 & 0 & 1 & 250 \end{array}\right)$

$\therefore x = 400, y = 350, z = 250$

c) $6x + 15y + 3z = 3850$

$4x + y + 5z = 3200$

$x + y + z = 1000$

$$\begin{pmatrix} 6 & 15 & 3 \\ 4 & 1 & 5 \\ 1 & 1 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3850 \\ 3200 \\ 1000 \end{pmatrix}$$

$$Let\ A = \begin{pmatrix} 6 & 15 & 3 \\ 4 & 1 & 5 \\ 1 & 1 & 1 \end{pmatrix}$$

$$|A| = \begin{vmatrix} 6 & 15 & 3 \\ 4 & 1 & 5 \\ 1 & 1 & 1 \end{vmatrix}$$

$$= (1)\begin{vmatrix} 15 & 3 \\ 1 & 5 \end{vmatrix} - (1)\begin{vmatrix} 6 & 3 \\ 4 & 5 \end{vmatrix} + (1)\begin{vmatrix} 6 & 15 \\ 4 & 1 \end{vmatrix}$$

$$= (75 - 3) - (30 - 12) + (6 - 60)$$

$$= 0$$

$Since\ |A| = 0, A\ is\ singular. thus\ the\ system\ of\ linear\ equation\ have\ no\ unique\ solution.$

$\therefore It\ is\ not\ possible\ for\ company\ P\ to\ award\ 15\ employees\ for\ their\ creativity$

$instead\ of\ 2\ with\ the\ same\ toal\ amount\ of\ money.$

10. (a) Determine wheather $f(x) = \frac{1}{x-4}$ and $g(x) = \frac{4x+1}{x}$ are inverse function of each other by computing their composite functions.

(b) Given $f(x) = \ln(1-3x)$.

(i) Determine the domain and range of $f(x)$. Then sketch the graph of $f(x)$.
(ii) Find $f^{-1}(\mathrm{x})$, if it exists. Hence, state the domain and range of $f^{-1}(\mathrm{x})$.

SOLUTION

a) $f(x) = \frac{1}{x-4}$ $\qquad g(x) = \frac{4x+1}{x}$

$$f[g(x)] = f\left[\frac{4x+1}{x}\right]$$

$$= \frac{1}{\left(\frac{4x+1}{x}\right) - 4}$$

$$= \frac{1}{\frac{4x+1-4x}{x}}$$

$$= x$$

$Since\ f[g(x)] = x, thus\ f\ and\ g\ are\ inverse\ function\ of\ each\ other.$

b) $f(x) = \ln(1-3x)$

i. $D_f : 1 - 3x > 0$

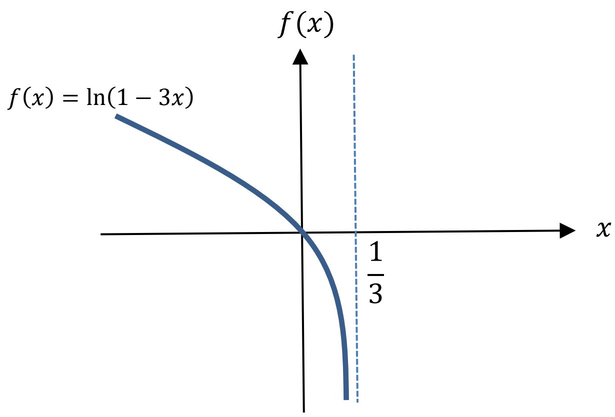
$$1 > 3x$$

$$3x < 1$$

$$x < \frac{1}{3}$$

$$D_f = \left\{x : x < \frac{1}{3}\right\}$$

$$R_f = \{y \in \mathbb{R}\}$$



ii. $f[f^{-1}(x)] = x$

$\ln[1 - 3f^{-1}(x)] = x$

$1 - 3f^{-1}(x) = e^x$

$3f^{-1}(x) = 1 - e^x$

$f^{-1}(x) = \dfrac{1 - e^x}{3}$

$D_{f^{-1}} = \{x \in \mathbb{R}\}$

$R_{f^{-1}} = \left\{y : y < \dfrac{1}{3}\right\}$