

SULIT
QS025/1
Mathematics
Paper 1
Semester II
Session 2017/2018
2 hours

QS025/1
Matematik
Kertas 1
Semester II
Sesi 2017/2018
2 jam



**KEMENTERIAN
PENDIDIKAN
MALAYSIA**

BAHAGIAN MATRIKULASI
MATRICULATION DIVISION

PEPERIKSAAN SEMESTER PROGRAM MATRIKULASI
MATRICULATION PROGRAMME EXAMINATION

MATEMATIK

Kertas 1

2 jam

JANGAN BUKA KERTAS SOALAN INI SEHINGGA DIBERITAHU.
DO NOT OPEN THIS QUESTION PAPER UNTIL YOU ARE TOLD TO DO SO.

CHOW CHOON WOOL
Kolej Matrikulasi Kedah
Kementerian Pendidikan Malaysia

Kertas soalan ini mengandungi **11** halaman bercetak.

This question paper consists of 11 printed pages.

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INSTRUCTIONS TO CANDIDATE:

This question paper consists of **10** questions.

Answer **all** questions.

All answers must be written in the answer booklet provided. Use a new page for each question.

The full marks for each question or section are shown in the bracket at the end of the question or section.

All steps must be shown clearly.

Only non-programmable scientific calculators can be used.

Numerical answers may be given in the form of π , e , surd, fractions or up to three significant figures, where appropriate, unless stated otherwise in the question.

ARAHAN KEPADA CALON:

*Kertas soalan ini mengandungi **10** soalan.*

*Jawab **semua** soalan.*

Semua jawapan hendaklah ditulis pada buku jawapan yang disediakan. Gunakan muka surat baharu bagi nombor soalan yang berbeza.

Markah penuh yang diperuntukkan bagi setiap soalan atau bahagian soalan ditunjukkan dalam kurungan pada penghujung soalan atau bahagian soalan.

Semua langkah kerja hendaklah ditunjukkan dengan jelas.

Kalkulator saintifik yang tidak boleh diprogramkan sahaja yang boleh digunakan.

Jawapan berangka boleh diberi dalam bentuk π , e , surd, pecahan atau sehingga tiga angka bererti, di mana-mana yang sesuai, kecuali jika dinyatakan dalam soalan.

LIST OF MATHEMATICAL FORMULAE
SENARAI RUMUS MATEMATIK

Trigonometry
Trigonometri

$$\sin (A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos (A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan (A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin A + \sin B = 2 \sin \frac{A+B}{2} \cos \frac{A-B}{2}$$

$$\sin A - \sin B = 2 \cos \frac{A+B}{2} \sin \frac{A-B}{2}$$

$$\cos A + \cos B = 2 \cos \frac{A+B}{2} \cos \frac{A-B}{2}$$

$$\cos A - \cos B = -2 \sin \frac{A+B}{2} \sin \frac{A-B}{2}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\begin{aligned} \cos 2A &= \cos^2 A - \sin^2 A \\ &= 2 \cos^2 A - 1 \\ &= 1 - 2 \sin^2 A \end{aligned}$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

$$\sin^2 A = \frac{1 - \cos 2A}{2}$$

$$\cos^2 A = \frac{1 + \cos 2A}{2}$$

$$\cos^2 x + \sin^2 x = 1$$

$$1 + \tan^2 x = \sec^2 x$$

$$\cot^2 x + 1 = \operatorname{cosec}^2 x$$

LIST OF MATHEMATICAL FORMULAE
SENARAI RUMUS MATEMATIK

Differentiation and Integration
Pembezaan dan Pengamiran

$$\frac{d}{dx}(\sin x) = \cos x$$

$$\frac{d}{dx}(\cos x) = -\sin x$$

$$\frac{d}{dx}(\tan x) = \sec^2 x$$

$$\frac{d}{dx}(\cot x) = -\operatorname{cosec}^2 x$$

$$\frac{d}{dx}(\sec x) = \sec x \tan x$$

$$\frac{d}{dx}(\operatorname{cosec} x) = -\operatorname{cosec} x \cot x$$

$$\int f'(x)e^{f(x)} dx = e^{f(x)} + c$$

$$\int \frac{f'(x)}{f(x)} dx = \ln |f(x)| + c$$

$$\int f'(x)[f(x)]^n dx = \frac{[f(x)]^{n+1}}{n+1} + c, n \neq -1$$

$$\int u dv = uv - \int v du$$

LIST OF MATHEMATICAL FORMULAE
SENARAI RUMUS MATEMATIK

Numerical Methods***Kaedah Berangka*****Newton-Raphson Method:*****Kaedah Newton-Raphson:***

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}, \quad n = 1, 2, 3, \dots$$

Trapezoidal Rule:***Petua Trapezium:***

$$\int_a^b f(x) dx \approx \frac{h}{2} [(y_0 + y_n) + 2(y_1 + y_2 + \dots + y_{n-1})], \quad h = \frac{b-a}{n}$$

Conics***Keratan Kon*****Circle:*****Bulatan:***

$$(x-h)^2 + (y-k)^2 = r^2$$

$$x^2 + y^2 + 2gx + 2fy + c = 0$$

$$xx_1 + yy_1 + g(x+x_1) + f(y+y_1) + c = 0$$

$$r = \sqrt{f^2 + g^2 - c}$$

$$d = \frac{|ah + bk + c|}{\sqrt{a^2 + b^2}}$$

LIST OF MATHEMATICAL FORMULAE
SENARAI RUMUS MATEMATIK

Parabola:*Parabola:*

$$(x - h)^2 = 4p(y - k)$$

$$(y - k)^2 = 4p(x - h)$$

$$F(h + p, k)$$

$$F(h, k + p)$$

Ellipse:*Elips:*

$$\frac{(x - h)^2}{a^2} + \frac{(y - k)^2}{b^2} = 1$$

$$F(h \pm c, k)$$

$$F(h, k \pm c)$$

Vectors*Vektor***Line and Plane:***Garis dan Satah:*

$$\vec{r} = \vec{a} + t\vec{v}$$

$$\vec{r} \cdot \vec{n} = \vec{a} \cdot \vec{n}$$

1 Evaluate

Nilaikan

$$\int_0^{\frac{\pi}{6}} \tan 2\theta \cos^2 2\theta \, d\theta.$$

[6 marks]

[6 markah]

2 Given vectors $\mathbf{p} = 3\mathbf{i} - 6\mathbf{j} + \alpha\mathbf{k}$ and $\mathbf{q} = \beta\mathbf{i} - 4\mathbf{j} + 5\mathbf{k}$ where α and β are constants.

Diberi vektor $\mathbf{p} = 3\mathbf{i} - 6\mathbf{j} + \alpha\mathbf{k}$ dan $\mathbf{q} = \beta\mathbf{i} - 4\mathbf{j} + 5\mathbf{k}$ dengan α dan β adalah pemalar.

(a) Find the values of α and β if \mathbf{p} and \mathbf{q} are parallel.

Cari nilai α dan β jika \mathbf{p} dan \mathbf{q} adalah selari.

[3 marks]

[3 markah]

(b) Given $\alpha = 1$, find β if \mathbf{p} and \mathbf{q} are perpendicular.

Diberi $\alpha = 1$, cari β jika \mathbf{p} dan \mathbf{q} adalah serenjang.

[3 marks]

[3 markah]

3 Given three points $P(-3, 2, -1)$, $Q(-2, 4, 5)$ and $R(1, -2, 4)$. Calculate the area of triangle PQR .

Diberi tiga titik $P(-3, 2, -1)$, $Q(-2, 4, 5)$ dan $R(1, -2, 4)$. Kira luas segitiga PQR .

[6 marks]

[6 markah]

- 4 Determine the vertices and foci of the ellipse $25x^2 + 4y^2 - 250x - 16y + 541 = 0$.
Sketch the ellipse and label the foci, center and vertices.

Tentukan bucu dan fokus bagi elips $25x^2 + 4y^2 - 250x - 16y + 541 = 0$. Lakarkan elips tersebut dan labelkan fokus, pusat dan bucu.

[7 marks]

[7 markah]

- 5 Show that the equation $\ln x + x - 4 = 0$ has a root between 1 and 3.

From the Newton-Raphson formula, show that iterative equation of the root is

$$x_{n+1} = \frac{x_n(5 - \ln x_n)}{1 + x_n}. \text{ Hence, if the initial value is } x_1 = 2, \text{ calculate the root correct to}$$

three decimal places.

Tunjukkan persamaan $\ln x + x - 4 = 0$ mempunyai punca antara 1 dan 3.

Daripada rumus Newton-Raphson, tunjukkan persamaan lelaran bagi punca tersebut

$$\text{adalah } x_{n+1} = \frac{x_n(5 - \ln x_n)}{1 + x_n}. \text{ Seterusnya, jika nilai awal adalah } x_1 = 2, \text{ hitungkan}$$

punca tersebut betul kepada tiga tempat perpuluhan.

[11 marks]

[11 markah]

- 6 Show that the line $2y - 5x + 4 = 0$ does not intersect the circle

$$x^2 + y^2 + 3x - 2y + 2 = 0. \text{ Find centre and radius of the circle.}$$

Hence, determine the shortest distance between the line and the circle.

Tunjukkan garis $2y - 5x + 4 = 0$ tidak menyilangi bulatan $x^2 + y^2 + 3x - 2y + 2 = 0$.

Cari pusat dan jejari bulatan tersebut.

Seterusnya, tentukan jarak terdekat antara garis dan bulatan tersebut.

[12 marks]

[12 markah]

- 7 Given the line $L: \frac{x-1}{2} = \frac{y-3}{-1} = \frac{z-2}{-3}$ and the planes $\pi_1: 2x - y - 2z = 17$ and $\pi_2: -4x - 3y + 5z = 10$.

Diberi garis $L: \frac{x-1}{2} = \frac{y-3}{-1} = \frac{z-2}{-3}$ dan satah $\pi_1: 2x - y - 2z = 17$ dan $\pi_2: -4x - 3y + 5z = 10$.

Find

Cari

- (a) the intersection point between L and π_1 .

titik persilangan antara L dan π_1 .

[4 marks]

[4 markah]

- (b) the acute angle between π_1 and π_2 .

sudut tirus antara π_1 dan π_2 .

[5 marks]

[5 markah]

- (c) the parametric equations of the line that passes through the point $(2, -1, 3)$ and perpendicular to the plane π_2 .

persamaan berparameter bagi garis melalui titik $(2, -1, 3)$ dan berserenjang dengan satah π_2 .

[3 marks]

[3 markah]

8 Express $\frac{6x^2 - x + 7}{(4 - 3x)(1 + x)^2}$ in partial fractions.

Hence, show that $\int_0^1 \frac{6x^2 - x + 7}{(4 - 3x)(1 + x)^2} dx = 1 + \ln 2$.

Ungkapkan $\frac{6x^2 - x + 7}{(4 - 3x)(1 + x)^2}$ dalam pecahan separa.

Seterusnya, tunjukkan $\int_0^1 \frac{6x^2 - x + 7}{(4 - 3x)(1 + x)^2} dx = 1 + \ln 2$.

[12 marks]

[12 markah]

9 (a) Find the general solution of the differential equation $\frac{dy}{dx} = y^2 x e^{-2x}$.

Give your answer in the form $y = f(x)$.

Cari penyelesaian am bagi persamaan pembezaan $\frac{dy}{dx} = y^2 x e^{-2x}$.

Beri jawapan anda dalam bentuk $y = f(x)$.

[6 marks]

[6 markah]

(b) Find the particular solution of the differential equation $\frac{dy}{dx} + \frac{xy}{1 + x^2} = \sqrt{1 + x^2}$,

given that $y = 1$ when $x = 0$.

Cari penyelesaian khusus bagi persamaan pembezaan $\frac{dy}{dx} + \frac{xy}{1 + x^2} = \sqrt{1 + x^2}$,

diberi $y = 1$ apabila $x = 0$.

[7 marks]

[7 markah]

10 Given the curve $y^2 = x$ and the line $y = -2x + 1$.

Diberi lengkung $y^2 = x$ dan garis $y = -2x + 1$.

(a) Determine the points of intersection between the curve and the line.

Tentukan titik persilangan antara lengkung dan garis tersebut.

[3 marks]

[3 markah]

(b) Sketch the curve and the line on the same axes. Shade the region R bounded by the curve and the line. Label the points of intersection.

Lakarkan lengkung dan garis tersebut pada paksi yang sama. Lorekkan rantau R yang dibatasi oleh lengkung dan garis tersebut. Labelkan titik-titik persilangannya.

[3 marks]

[3 markah]

(c) Find the area of the region R .

Cari luas bagi rantau R .

[4 marks]

[4 markah]

(d) Calculate the volume of the solid generated when the region R is rotated 2π radians about the y -axis.

Hitung isipadu bongkah yang terjana apabila rantau R diputarakan 2π radian pada paksi- y .

[5 marks]

[5 markah]