



QS 025/1

**Matriculation Programme
Examination**

Semester I

Session 2016/2017

1. Find the angle between the line $l: \langle x, y, z \rangle = \langle 1, 3, -1 \rangle + t\langle 2, 1, 0 \rangle$ and the plane $\Pi: 3x - 2y + z = 5$.

SOLUTION

$$\underline{v} = \langle 2, 1, 0 \rangle$$

$$\underline{n} = \langle 3, -2, 1 \rangle$$

$$\underline{v} \cdot \underline{n} = \langle 2, 1, 0 \rangle \cdot \langle 3, -2, 1 \rangle$$

$$= (2)(3) + (1)(-2) + (0)(1)$$

$$= 6 - 2 + 0$$

$$= 4$$

$$|\underline{v}| = \sqrt{2^2 + 1^2 + 0^2}$$

$$= \sqrt{5}$$

$$|\underline{n}| = \sqrt{3^2 + (-2)^2 + 1^2}$$

$$= \sqrt{14}$$

$$\cos \theta = \frac{\underline{v} \cdot \underline{n}}{|\underline{v}| \cdot |\underline{n}|}$$

$$= \frac{4}{\sqrt{5} \cdot \sqrt{14}}$$

$$= 0.4781$$

$$\theta = \cos^{-1} 0.4781$$

$$= 61.4^\circ \text{ (Angle between the line } l \text{ and the normal vector of plane } \Pi)$$

$$\text{Angle between the line } l \text{ and the vector of plane } \Pi = 90^\circ - 61.4^\circ = 28.6^\circ$$

2. Solve $\int \frac{e^{2x}}{1-e^{2x}} dx$.

SOLUTION

$$\int \frac{e^{2x}}{1-e^{2x}} dx$$

$$u = 1 - e^{2x}$$

$$\frac{du}{dx} = -2e^{2x}$$

$$du = -2e^{2x} dx$$

$$e^{2x} dx = -\frac{du}{2}$$

$$\begin{aligned} \int \frac{e^{2x}}{1-e^{2x}} dx &= -\frac{1}{2} \int \frac{1}{u} du \\ &= -\frac{1}{2} \ln|u| + c \\ &= -\frac{1}{2} \ln|1 - e^{2x}| + c \end{aligned}$$

Alternative

$$\int \frac{e^{2x}}{1-e^{2x}} dx$$

$$u = e^{2x}$$

$$\frac{du}{dx} = 2e^{2x}$$

$$du = 2e^{2x} dx$$

$$e^{2x} dx = \frac{1}{2} du$$

$$\int \frac{e^{2x}}{1-e^{2x}} dx = \int \frac{1}{1-u} \left(\frac{1}{2} du \right)$$

$$\begin{aligned} &= \frac{1}{2} \int \frac{1}{1-u} du \\ &= -\frac{1}{2} \ln|1-u| + c \\ &= -\frac{1}{2} \ln|1-e^{2x}| + c \end{aligned}$$

3. Given four points $A = (-2, -8, 4)$, $B(2, -\omega, -1)$, $C = (0, -9, 0)$ and $D = (-4, -3, 7)$.
Determine the value of ω if $\overrightarrow{AB} \cdot (\overrightarrow{AC} \times \overrightarrow{AD}) = 64$.

SOLUTION

$$A = (-2, -8, 4), B(2, -\omega, -1), C = (0, -9, 0) \text{ and } D = (-4, -3, 7)$$

$$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$$

$$= \langle 2, -\omega, -1 \rangle - \langle -2, -8, 4 \rangle$$

$$= \langle 4, 8 - \omega, -5 \rangle$$

$$\overrightarrow{AC} = \overrightarrow{OC} - \overrightarrow{OA}$$

$$= \langle 0, -9, 0 \rangle - \langle -2, -8, 4 \rangle$$

$$= \langle 2, -1, -4 \rangle$$

$$\overrightarrow{AD} = \overrightarrow{OD} - \overrightarrow{OA}$$

$$= \langle -4, -3, 7 \rangle - \langle -2, -8, 4 \rangle$$

$$= \langle -2, 5, 3 \rangle$$

$$\overrightarrow{AC} \times \overrightarrow{AD} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & -1 & -4 \\ -2 & 5 & 3 \end{vmatrix}$$

$$= 17\mathbf{i} + 2\mathbf{j} + 8\mathbf{k}$$

$$= \langle 17, 2, 8 \rangle$$

$$\overrightarrow{AB} \cdot (\overrightarrow{AC} \times \overrightarrow{AD}) = 64$$

$$\langle 4, 8 - \omega, -5 \rangle \cdot \langle 17, 2, 8 \rangle = 64$$

$$(4)(17) + (8 - \omega)(2) + (-5)(8) = 64$$

$$68 + 16 - 2\omega - 40 = 64$$

$$44 - 2\omega = 64$$

$$2\omega = -20$$

$$\omega = -10$$

4. Find the vertex, focus and directrix for the parabola $y^2 + 64 = 8y - 16x$. Hence, sketch and label the vertex, focus and directrix for the curve.

SOLUTION

$$y^2 + 64 = 8y - 16x$$

$$y^2 - 8y = -16x - 64$$

$$y^2 - 8y + \left(\frac{-8}{2}\right)^2 = -16x - 64 + \left(\frac{-8}{2}\right)^2$$

$$(y - 4)^2 = -16x - 64 + 16$$

$$(y - 4)^2 = -16x - 48$$

$$(y - 4)^2 = -16(x + 3)$$

Compare

$$(y - 4)^2 = -16(x + 3)$$

$$(y - k)^2 = 4p(x - h)$$

$$h = -3, \quad k = 4, \quad 4p = -16 \implies p = -4$$

Vertex

$$V(h, k) = V(-3, 4)$$

Focus,

$$F(h + p, k) = F(-3 - 4, 4)$$

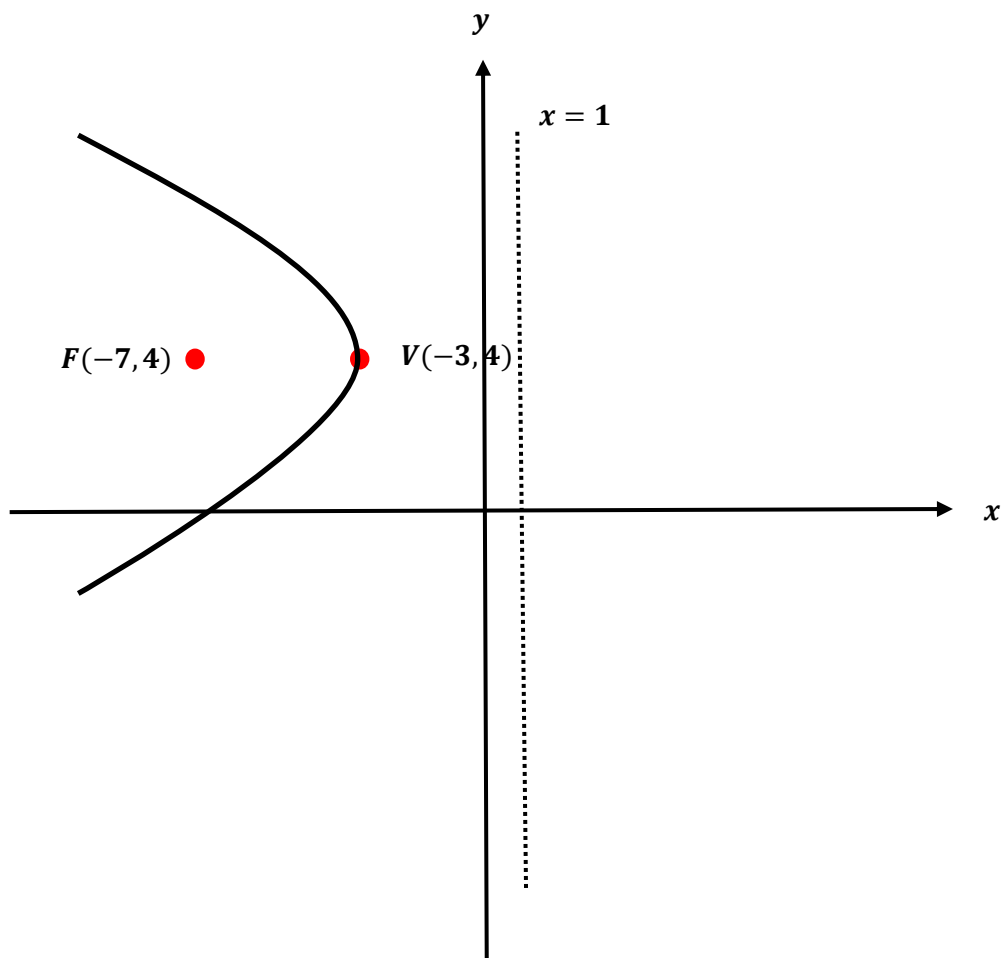
$$= F(-7, 4)$$

Directrix,

$$x = h - p$$

$$= -3 + 4$$

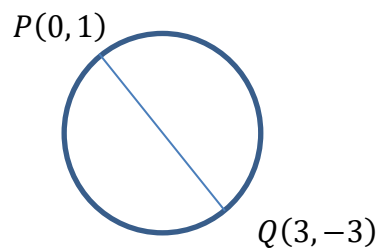
$$x = 1$$



5. The end points of the diameter of a circle are $P(0, 1)$ and $Q(3, -3)$.
- Determine an equation of the circle.
 - Find an equation of the tangent line to the circle at the point $P(0, 1)$.

SOLUTION

- a) Diameter: $P(0, 1)$ and $Q(3, -3)$



$$\begin{aligned} \text{Center, } C(h, k) &= \left(\frac{0+3}{2}, \frac{1-3}{2} \right) \\ &= \left(\frac{3}{2}, -1 \right) \end{aligned}$$

$$\begin{aligned} \text{Diameter} &= \sqrt{(3-0)^2 + (-3-1)^2} \\ &= \sqrt{25} \\ &= 5 \end{aligned}$$

$$\begin{aligned} \text{Radius, } r &= \frac{1}{2}(\text{diameter}) \\ &= \frac{5}{2} \end{aligned}$$

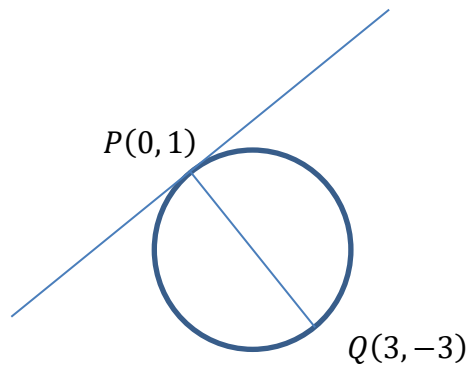
Standard equation of circle

$$(x - h)^2 + (y - k)^2 = r^2$$

$$\left(x - \frac{3}{2}\right)^2 + (y + 1)^2 = \left(\frac{5}{2}\right)^2$$

$$\left(x - \frac{3}{2}\right)^2 + (y + 1)^2 = \frac{25}{4}$$

b)

**Slope of PQ**

$$\begin{aligned} m_{pq} &= \frac{-3 - 1}{3 - 0} \\ &= -\frac{4}{3} \end{aligned}$$

Slope of tangent line

$$\begin{aligned} m_t &= -\frac{1}{m_{pq}} \\ &= -\frac{1}{-\frac{4}{3}} \\ &= \frac{3}{4} \end{aligned}$$

Equation of the tangent line to the circle at the point $P(0, 1)$

$$y - y_1 = m_t(x - x_1)$$

$$y - 1 = \frac{3}{4}(x - 0)$$

$$y = \frac{3}{4}x + 1$$

6. In a Chemistry experiment, sodium hydroxide, NaOH, reacts with hydrochloric acid, HCl, to form sodium chloride salt, NaCl, and water. Before the reaction starts, no NaCl salt is formed. At time t (minute), the mass of NaCl salt formed is x grams and the rate of change of x is given by $\frac{dx}{dt} = \alpha(50 - x)$, where α is a positive constant.
- Find the general solution for the above equation.
 - Find the particular solution if 35 grams of NaCl salt has formed in the first 30 minutes.
 - Hence, find
 - The mass of NaCl salt formed in 60 minutes.
 - The time taken to form 40 grams of NaCl salt.

SOLUTION

a) $\frac{dx}{dt} = \alpha(50 - x)$

$$\frac{1}{(50 - x)} dx = \alpha dt$$

$$\int \frac{1}{(50 - x)} dx = \int \alpha dt$$

$$-\ln(50 - x) = \alpha t + c$$

$$\ln(50 - x) = -(\alpha t + c)$$

$$50 - x = e^{-(\alpha t + c)}$$

$$50 - x = e^{-\alpha t} \cdot e^{-c}$$

$$50 - x = Ae^{-\alpha t}; \quad A = e^{-c}$$

When $t = 0$, $x = 0$

$$50 - 0 = Ae^{-0}$$

$$A = 50$$

General Solution

$$50 - x = 50e^{-\alpha t}$$

$$50 - 50e^{-\alpha t} = x$$

$$x = 50(1 - e^{-\alpha t})$$

b) When $t = 30$, $x = 35$

$$35 = 50(1 - e^{-30\alpha})$$

$$1 - e^{-30\alpha} = \frac{35}{50}$$

$$1 - e^{-30\alpha} = 0.7$$

$$e^{-30\alpha} = 1 - 0.7$$

$$e^{-30\alpha} = 0.3$$

$$\ln e^{-30\alpha} = \ln 0.3$$

$$-30\alpha = -1.2040$$

$$\alpha = \frac{-1.2040}{-30}$$

$$\alpha = 0.0401$$

Particular

$$x = 50(1 - e^{-0.0401t})$$

d) i) When $t = 60$

$$\begin{aligned}x &= 50[1 - e^{-0.0401(60)}] \\&= 50[1 - e^{-0.0401(60)}] \\&= 45.5\end{aligned}$$

ii) When $x = 40$

$$40 = 50(1 - e^{-0.0401t})$$

$$\frac{40}{50} = 1 - e^{-0.0401t}$$

$$0.8 = 1 - e^{-0.0401t}$$

$$e^{-0.0401t} = 1 - 0.8$$

$$e^{-0.0401t} = 0.2$$

$$\ln e^{-0.0401t} = \ln 0.2$$

$$-0.0401t = \ln 0.2$$

$$t = \frac{\ln 0.2}{-0.0401}$$

$$t = 40.1$$

7. (a) Show that equation $-4x^2 + 5x + 7 = 0$ has a root on the interval $[-2, 0]$. Use the Newton-Raphson method to find the root of the equation correct to four decimal places.
- (b) Estimate the value of $\int_{-\pi}^0 x \cos x \, dx$ using trapezoidal rule with subinterval $\frac{\pi}{4}$. Give your answer correct to four decimal places.

SOLUTION

a) $-4x^2 + 5x + 7 = 0$

$$f(x) = -4x^2 + 5x + 7$$

When $x = -2$

$$\begin{aligned} f(-2) &= -4(-2)^2 + 5(-2) + 7 \\ &= -16 - 10 + 7 \\ &= -19 < 0 \end{aligned}$$

When $x = 0$

$$\begin{aligned} f(0) &= -4(0)^2 + 5(0) + 7 \\ &= 7 > 0 \end{aligned}$$

Since $f(-2) < 0$ and $f(0) > 0$, therefore

equation $-4x^2 + 5x + 7 = 0$ has a root on the interval $[-2, 0]$

Newton-Raphson Method

$$f(x) = -4x^2 + 5x + 7$$

$$f'(x) = -8x + 5$$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$x_{n+1} = x - \frac{-4x^2 + 5x + 7}{-8x + 5}$$

$$x_{n+1} = x - \frac{4x^2 - 5x - 7}{8x - 5}$$

$$x_0 = -1$$

$$x_1 = -1 - \frac{4(-1)^2 - 5(-1) - 7}{8(-1) - 5} = -0.8462$$

$$x_2 = -0.8462 - \frac{4(-0.8462)^2 - 5(-0.8462) - 7}{8(-0.8462) - 5} = -0.8381$$

$$x_3 = -0.8381 - \frac{4(-0.8381)^2 - 5(-0.8381) - 7}{8(-0.8381) - 5} = -0.8381$$

$$b) \int_{-\pi}^0 x \cos x \, dx$$

$$h = \frac{\pi}{4}$$

x	$f(x) = x \cos x$	
	First and last ordinate	Remaining ordinat
$-\pi$	3.14159	
$-\frac{3\pi}{4}$		1.66608
$-\frac{2\pi}{4}$		0
$-\frac{\pi}{4}$		-0.55536
0	0	
Total	3.14159	1.11072

$$\int_a^b f(x) \, dx = \frac{h}{2} [(y_0 + y_n) + 2(y_1 + y_2 + \dots + y_{n-1})]$$

$$\int_{-\pi}^0 x \cos x \, dx = \frac{\pi}{2(4)} [(3.14159) + 2(1.11072)]$$

$$= \frac{\pi}{2(4)} [(3.14159) + 2(1.11072)]$$

$$= 2.1061$$

8. Given the curve $y = 4x^2$ and the line $y = 6x$.
- Find the intersection points.
 - Sketch the region enclosed by the curve and the line.
 - Calculate the area of the region enclosed by the curve and the line.
 - Calculate the volume of the solid generated when the region is revolved completely about the y-axis.

SOLUTION

a) $y = 4x^2$ (1)

$$y = 6x \text{ (2)}$$

Substitute (2) into (1)

$$6x = 4x^2$$

$$4x^2 - 6x = 0$$

$$2x(2x - 3) = 0$$

$$2x = 0 \quad 2x - 3 = 0$$

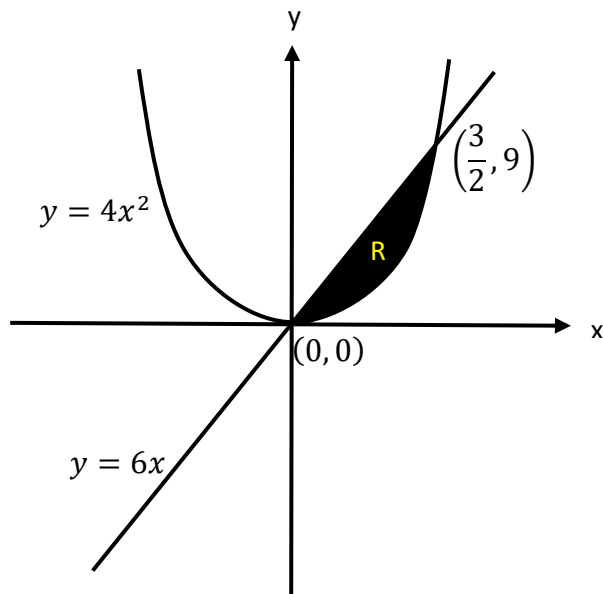
$$x = 0 \quad x = \frac{3}{2}$$

$$\text{When } x = 0 \quad \rightarrow \quad y = 6(0) = 0 \quad \rightarrow (0, 0)$$

$$\text{When } x = \frac{3}{2} \quad \rightarrow \quad y = 6\left(\frac{3}{2}\right) = 9 \quad \rightarrow \left(\frac{3}{2}, 9\right)$$

\therefore *Intersection points: (0, 0) and $\left(\frac{3}{2}, 9\right)$*

b)



$$\text{c) Area, } R = \int_0^{\frac{3}{2}} 6x - 4x^2 \, dx$$

$$= \left[\frac{6x^2}{2} - \frac{4x^3}{3} \right]_0^{\frac{3}{2}}$$

$$= \left[3x^2 - \frac{4x^3}{3} \right]_0^{\frac{3}{2}}$$

$$= \left[3 \left(\frac{3}{2} \right)^2 - \frac{4 \left(\frac{3}{2} \right)^3}{3} \right] - \left[3(0)^2 - \frac{4(0)^3}{3} \right]$$

$$= \left[3 \left(\frac{9}{4} \right) - \frac{4 \left(\frac{27}{8} \right)}{3} \right] - [0]$$

$$= \left[\frac{27}{4} - \frac{27}{6} \right]$$

$$= \frac{9}{4} \text{ unit}^2$$

$$d) y = 4x^2 \rightarrow x^2 = \frac{y}{4}$$

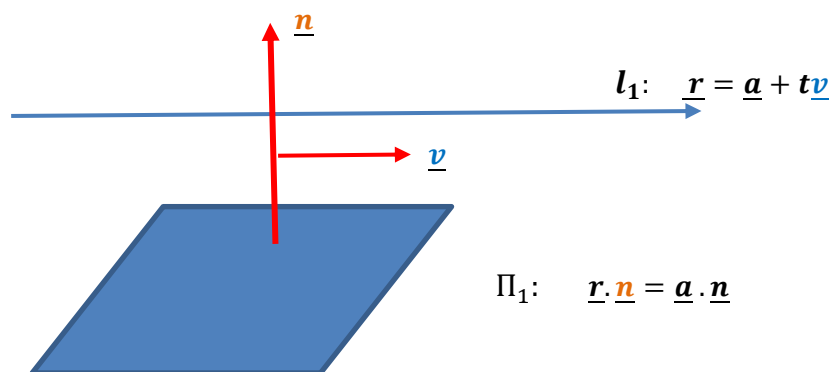
$$y = 6x \rightarrow x = \frac{y}{6} \rightarrow x^2 = \frac{y^2}{36}$$

$$\begin{aligned} \text{Volume, } V &= \pi \int_0^9 \frac{y}{4} - \frac{y^2}{36} dy \\ &= \pi \left[\frac{y^2}{4(2)} - \frac{y^3}{36(3)} \right]_0^9 \\ &= \pi \left[\frac{y^2}{8} - \frac{y^3}{108} \right]_0^9 \\ &= \pi \left[\left(\frac{9^2}{8} - \frac{9^3}{108} \right) - \left(\frac{0^2}{8} - \frac{0^3}{108} \right) \right] \\ &= \pi \left[\left(\frac{81}{8} - \frac{729}{108} \right) - 0 \right] \\ &= \pi \left(\frac{27}{8} \right) \\ &= \frac{27}{8} \pi \text{ unit}^3 \end{aligned}$$

9. (a) If the line $l_1: \langle x, y, z \rangle = \langle 1, 1, 2 \rangle + t\langle 2, -1, 3 \rangle$ does not intersect with the plane $\Pi_1: Ax + By + Cz = 0$, show that $2A - B + 3C = 0$. Hence, find the equation of plane Π_1 if the plane passes through the point $(1, 0, 1)$.
- (b) Given the line $l_2: x = x_0 + tv_1, y = y_0 + tv_2, z = z_0 + tv_3$, the plane $\Pi_2: x - y + 2z = 0$, and a point $(x_0, y_0, z_0) \neq (0, 0, 0)$ is on the plane.
- (i) If l_2 is perpendicular to the plane Π_2 , show that $\langle v_1, v_2, v_3 \rangle = v_2\langle -1, 1, -2 \rangle$; $v_2 \neq 0$.
- (ii) Give one example of the equation of straight line which satisfy part 9(b)(i)

SOLUTION

- a) A line does not intersect with a plane \rightarrow line is parallel to the plane \rightarrow
 \underline{n} is perpendicular to $\underline{v} \rightarrow \underline{n} \cdot \underline{v} = 0$ (See the diagram below)



$$l_1: \langle x, y, z \rangle = \langle 1, 1, 2 \rangle + t\langle 2, -1, 3 \rangle \quad \rightarrow \quad \underline{v} = \langle 2, -1, 3 \rangle$$

$$\Pi_1: Ax + By + Cz = 0, \quad \rightarrow \quad \underline{n} = \langle A, B, C \rangle$$

$$\underline{n} \cdot \underline{v} = 0$$

$$\langle A, B, C \rangle \cdot \langle 2, -1, 3 \rangle = 0$$

$$2A - B + 3C = 0$$

$$\Pi_1: Ax + By + Cz = 0$$

$$2A - B + 3C = 0 \quad \dots\dots\dots(1)$$

Point (1, 0, 1) is on the plane $Ax + By + Cz = 0$

$$A(1) + B(0) + C(1) = 0$$

$$A + C = 0$$

$$C = -A \quad \dots\dots\dots(2)$$

Substitute (2) into (1)

$$2A - B + 3(-A) = 0$$

$$2A - B - 3A = 0$$

$$-B - A = 0$$

$$B = -A \quad \dots\dots\dots(3)$$

$$\therefore Ax + By + Cz = 0 \quad \rightarrow \quad Ax + (-A)y + (-A)z = 0$$

$$Ax - Ay - Az = 0$$

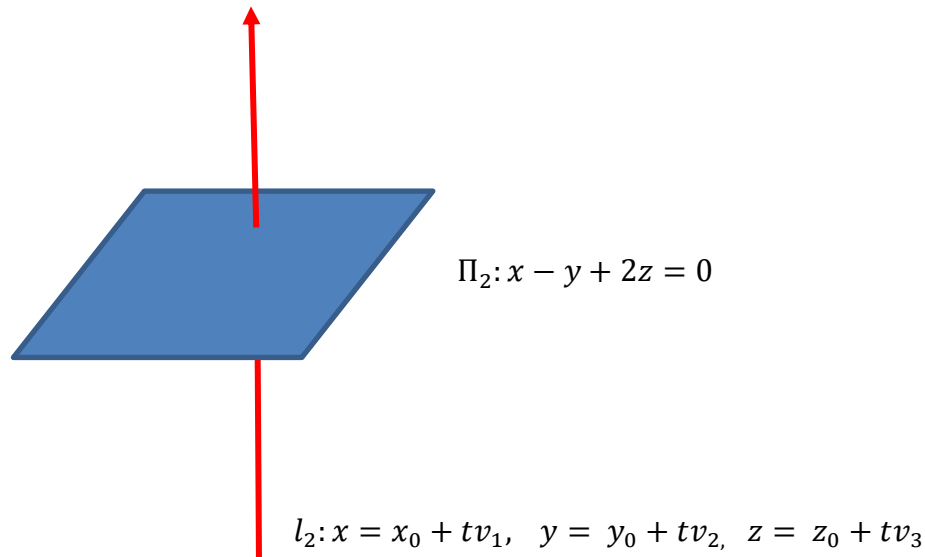
$$A(x - y - z) = 0$$

$$x - y - z = 0$$

b) line $l_2: x = x_0 + tv_1, y = y_0 + tv_2, z = z_0 + tv_3$

$$\Pi_2: x - y + 2z = 0$$

i)



l_2 is perpendicular to the plane $\Pi_2 \rightarrow \underline{v}$ is parallel to $\underline{n} \rightarrow \underline{v} \times \underline{n} = 0$

$$\underline{v} = \langle v_1, v_2, v_3 \rangle$$

$$\underline{n} = \langle 1, -1, 2 \rangle$$

$$\underline{v} \times \underline{n} = 0$$

$$\begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ v_1 & v_2 & v_3 \\ 1 & -1 & 2 \end{vmatrix} = 0$$

$$[2v_2 - (-v_3)]\underline{i} - (2v_1 - v_3)\underline{j} + (-v_1 - v_2)\underline{k} = 0$$

$$[2v_2 + v_3]\underline{i} - (2v_1 - v_3)\underline{j} + (-v_1 - v_2)\underline{k} = 0$$

$$2v_2 + v_3 = 0 \quad \rightarrow \quad v_3 = -2v_2$$

$$2v_1 - v_3 = 0$$

$$v_1 + v_2 = 0 \quad \rightarrow \quad v_1 = -v_2$$

$$\begin{aligned} \therefore \langle v_1, v_2, v_3 \rangle &= \langle -v_2, v_2, -2v_2 \rangle \\ &= v_2 \langle -1, 1, -2 \rangle; \quad v_2 \neq 0 \end{aligned}$$

ii) $\underline{v} = \underline{n} = \langle 1, -1, 2 \rangle$

At the point $(x_0, y_0, z_0) = (-1, 1, 1)$

Equation of straight line l_2 :

$$x = -1 + t(1) \quad \rightarrow \quad x = -1 + t$$

$$y = 1 + t(-1) \quad \rightarrow \quad y = 1 - t$$

$$z = 1 + t(2) \quad \rightarrow \quad z = 1 + 2t$$

10. (a) Show that the expression $\frac{4x^4+2x^2-1}{(2x-3)^2(x+1)}$ can be written as

$$x + 2 + \frac{A}{2x-3} + \frac{B}{(2x-3)^2} + \frac{C}{x+1}.$$

(b) From part 10(a), determine the values of A, B and C. Hence, solve

$$\int \frac{4x^4+2x^2-1}{(2x-3)^2(x+1)} dx$$

SOLUTION

a) Improper Fraction → Use long division

$$\begin{aligned} \frac{4x^4 + 2x^2 - 1}{(2x - 3)^2(x + 1)} &= \frac{4x^4 + 2x^2 - 1}{(4x^2 - 12x + 9)(x + 1)} \\ &= \frac{4x^4 + 2x^2 - 1}{4x^3 + 4x^2 - 12x^2 - 12x + 9x + 9} \\ &= \frac{4x^4 + 2x^2 - 1}{4x^3 - 8x^2 - 3x + 9} \end{aligned}$$

$$\begin{array}{r} \overline{) 4x^4 + 0x^3 + 2x^2 + 0x - 1} \\ \underline{4x^4 - 8x^3 - 3x^2 + 9x + 0} \\ 8x^3 + 5x^2 - 9x - 1 \\ \underline{8x^3 - 16x^2 - 6x + 18} \\ 21x^2 - 3x - 19 \end{array}$$

$$\begin{aligned}\frac{4x^4 + 2x^2 - 1}{(2x - 3)^2(x + 1)} &= x + 2 + \frac{21x^2 - 3x - 19}{(2x - 3)^2(x + 1)} \\ &= x + 2 + \frac{A}{2x - 3} + \frac{B}{(2x - 3)^2} + \frac{C}{x + 1}\end{aligned}$$

b)

$$\begin{aligned}\frac{21x^2 - 3x - 19}{(2x - 3)^2(x + 1)} &= \frac{A}{2x - 3} + \frac{B}{(2x - 3)^2} + \frac{C}{x + 1} \\ &= \frac{A(2x - 3)(x + 1) + B(x + 1) + C(2x - 3)^2}{(2x - 3)^2(x + 1)}\end{aligned}$$

$$21x^2 - 3x - 19 = A(2x - 3)(x + 1) + B(x + 1) + C(2x - 3)^2$$

When $x = -1$

$$21(-1)^2 - 3(-1) - 19 = C[2(-1) - 3]^2$$

$$5 = 25C$$

$$C = \frac{5}{25}$$

$$C = \frac{1}{5}$$

When $x = \frac{3}{2}$

$$21\left(\frac{3}{2}\right)^2 - 3\left(\frac{3}{2}\right) - 19 = B\left[\left(\frac{3}{2}\right) + 1\right]$$

$$21\left(\frac{9}{4}\right) - \frac{9}{2} - 19 = B\left[\frac{5}{2}\right]$$

$$21\left(\frac{9}{4}\right) - \frac{9}{2} - 19 = \frac{5}{2}B$$

$$\frac{95}{4} = \frac{5}{2}B$$

$$B = \frac{95}{4} \times \frac{2}{5}$$

$$B = \frac{19}{2}$$

When $x = 0$

$$-19 = A[2(0) - 3][(0) + 1] + B(0 + 1) + C[2(0) - 3]^2$$

$$-19 = -3A + B + 9C$$

$$-19 = -3A + \left(\frac{19}{2}\right) + 9\left(\frac{1}{5}\right)$$

$$-19 = -3A + \left(\frac{19}{2}\right) + \left(\frac{9}{5}\right)$$

$$-19 = -3A + \frac{113}{10}$$

$$3A = \frac{113}{10} + 19$$

$$3A = \frac{303}{10}$$

$$A = \frac{101}{10}$$

$$\therefore A = \frac{101}{10}; B = \frac{19}{2}; C = \frac{1}{5}$$

$$\frac{4x^4 + 2x^2 - 1}{(2x - 3)^2(x + 1)} = x + 2 + \frac{101}{10(2x - 3)} + \frac{19}{2(2x - 3)^2} + \frac{1}{5(x + 1)}$$

$$\begin{aligned} \int \frac{4x^4 + 2x^2 - 1}{(2x - 3)^2(x + 1)} dx &= \int x + 2 + \frac{101}{10(2x - 3)} + \frac{19}{2(2x - 3)^2} + \frac{1}{5(x + 1)} dx \\ &= \int (x + 2) dx + \frac{101}{10} \int \frac{1}{(2x - 3)} dx + \frac{19}{2} \int (2x - 3)^{-2} dx + \frac{1}{5} \int \frac{1}{(x + 1)} dx \\ &= \frac{x^2}{2} + 2x + \frac{101}{(10)2} \int \frac{2}{(2x - 3)} dx + \frac{19}{2} \left[\frac{(2x - 3)^{-1}}{(-1)(2)} \right] + \frac{1}{5} \ln|x + 1| + C \\ &= \frac{x^2}{2} + 2x + \frac{101}{20} \ln|2x - 3| dx - \frac{19}{4} \left[\frac{1}{(2x - 3)} \right] + \frac{1}{5} \ln|x + 1| + C \\ &= \frac{x^2}{2} + 2x + \frac{101}{20} \ln|2x - 3| dx - \frac{19}{4(2x - 3)} + \frac{1}{5} \ln|x + 1| + C \end{aligned}$$