

QS 015/1
Matriculation Programme
Examination
Semester I
Session 2015/2016

1. Evaluate the solution of $4^{y-2} = \frac{1}{3^{-y}}$ up to three decimal places.
2. The first three terms of a geometric sequence are $\left(\frac{4}{3}m - 2\right)$, $(2m - 1)$ and 12.
Determine the value of m . Hence, find the sixth term for this sequence.

3. Solve the equation

$$2 + \log_2 x = 15 \log_x 2.$$

4. (a) Determine the values of x so that $\begin{bmatrix} 1 & x & -1 \\ x & 0 & 1 \\ 1 & 3 & -1 \end{bmatrix}$ is singular.

(b) If $A = \begin{bmatrix} 3 & 5 \\ 1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 4 \\ 0 & -1 \end{bmatrix}$, find C when $A = BCB^{-1}$.

5. (a) Expand $(2 + x)^{-\frac{1}{2}}$ in ascending powers of x , up to the term x^3 .

(b) Use the expansion in (a) to approximate $\sqrt{\frac{2}{3}}$.

6. Given $z_1 = 3 - 3i$ and $z_2 = 3 + 2i$.

(a) Write \bar{z}_1 in polar form.

(b) Express $\frac{(\bar{z}_1 z_2)}{13} + \left[\frac{i^3}{-z_2} \right]$ in the form of $a + bi$, $c, b \in R$.

7. A curve $y = ax^2 + bx + c$ where a , b and c are constants, passes through the points $(2, 11)$, $(-1, -16)$ and $(3, 28)$.

(a) By using the above information, construct a system containing three linear equations.

- (b) Express the above system as a matrix equation $AX=B$.
- (c) Find the inverse of matrix A by using the adjoint matrix method. Hence, obtain the values of a, b and c.
8. Given a function $f(x) = \sqrt{3 - 2x}$.
- (a) Show that f is a one to one function.
- (b) Find the domain and range of f .
- (c) Determine the inverse function of f and state its domain and range.
- (d) Sketch the graphs of f and f^{-1} on the same axis.
9. (a) The function f is given as $f(x) = \frac{ax+2}{3x-4}$, $x \neq \frac{4}{3}$. If $(f \circ f)(x) = x$, find the value of a.
- (b) Let $f(x) = \ln|3x + 2|$ and $g(x) = e^{-x} + 2$ be two functions. Evaluate $(g \circ f)^{-1}(3)$.
10. (a) Solve the inequality $\left| \frac{x-1}{x+3} \right| > 2$.
- (b) Show that $\frac{2^x \times 4^{2x}}{8^x} = 2^{2x}$.
- Hence, find the interval for x so that $\frac{2^x \times 4^{2x}}{8^x} - 13(2^x) + 36 \geq 0$.

END OF QUESTION PAPER

1. Evaluate the solution of $4^{y-2} = \frac{1}{3^{-y}}$ up to three decimal places.

SOLUTION

$$4^{y-2} = \frac{1}{3^{-y}}$$

$$4^{y-2} = 3^y$$

$$\ln 4^{y-2} = \ln 3^y$$

$$(y - 2) \ln 4 = y \ln 3$$

$$y \ln 4 - 2 \ln 4 = y \ln 3$$

$$y \ln 4 - y \ln 3 = 2 \ln 4$$

$$y (\ln 4 - \ln 3) = 2 \ln 4$$

$$y = \frac{2 \ln 4}{(\ln 4 - \ln 3)}$$

$$= 9.638$$

2. The first three terms of a geometric sequence are $(\frac{4}{3}m - 2)$, $(2m - 1)$ and 12.

Determine the value of m . Hence, find the sixth term for this sequence.

SOLUTION

Geometric Sequence

If a, b, c are three consecutive terms,

$$b = \pm\sqrt{ac}$$

$$(\frac{4}{3}m - 2), (2m - 1), 12$$

$$(2m - 1) = \sqrt{(\frac{4}{3}m - 2)(12)}$$

$$(2m - 1) = \sqrt{16m - 24}$$

$$(2m - 1)^2 = 16m - 24$$

$$4m^2 - 4m + 1 = 16m - 24$$

$$4m^2 - 20m + 25 = 0$$

$$(2m - 5)(2m - 5) = 0$$

$$m = \frac{5}{2}$$

Alternative

$$r = \frac{T_2}{T_1} = \frac{T_3}{T_2}$$

$$\frac{2m - 1}{\frac{4}{3}m - 2} = \frac{12}{2m - 1}$$

$$(2m - 1)^2 = 16m - 24$$

$$4m^2 - 20m + 25 = 0$$

$$(2m - 5)(2m - 5) = 0$$

$$m = \frac{5}{2}$$

When $m = \frac{5}{2}$,

$$T_1 = a = \frac{4}{3}m - 2 = \frac{4}{3}\left(\frac{5}{2}\right) - 2 = \frac{4}{3}$$

$$T_2 = 2m - 1 = 2\left(\frac{5}{2}\right) - 1 = 4$$

$$r = \frac{T_2}{T_1}$$

$$r = \frac{4}{\frac{4}{3}} = 3$$

$$T_n = ar^{n-1}$$

$$T_6 = \left(\frac{4}{3}\right)(3)^5$$

$$= 324$$

3. Solve the equation

$$2 + \log_2 x = 15 \log_x 2.$$

SOLUTION

$$2 + \log_2 x = 15 \log_x 2$$

$$2 + \log_2 x = 15 \left(\frac{\log_2 2}{\log_2 x} \right)$$

$$2 + \log_2 x = 15 \left(\frac{1}{\log_2 x} \right)$$

$$2 + \log_2 x = \frac{15}{\log_2 x}$$

Let $u = \log_2 x$

$$2 + u = \frac{15}{u}$$

$$2u + u^2 = 15$$

$$u^2 + 2u - 15 = 0$$

$$(u - 3)(u + 5) = 0$$

$$u = 3$$

$$\log_2 x = 3$$

$$x = 2^3$$

$$x = 8$$

$$u = -5$$

$$\log_2 x = -5$$

$$x = 2^{-5}$$

$$x = \frac{1}{32}$$

$$\log_b a = \frac{\log_c a}{\log_c b}$$

$$\log_b a = \frac{\log_c a}{\log_c b}$$

4. (a) Determine the values of x so that $\begin{bmatrix} 1 & x & -1 \\ x & 0 & 1 \\ 1 & 3 & -1 \end{bmatrix}$ is singular.

(b) If $A = \begin{bmatrix} 3 & 5 \\ 1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 4 \\ 0 & -1 \end{bmatrix}$, find C when $A = BCB^{-1}$.

SOLUTION

(a)

$$\begin{vmatrix} 1 & x & -1 \\ x & 0 & 1 \\ 1 & 3 & -1 \end{vmatrix} = 0$$

$$(1) \begin{vmatrix} 0 & 1 \\ 3 & -1 \end{vmatrix} - (x) \begin{vmatrix} x & 1 \\ 1 & -1 \end{vmatrix} + (-1) \begin{vmatrix} x & 0 \\ 1 & 3 \end{vmatrix} = 0$$

$$(1)[0 - 3] - (x)[-x - 1] + (-1)[3x - 0] = 0$$

$$-3 + x^2 + x - 3x = 0$$

$$x^2 - 2x - 3 = 0$$

$$(x - 3)(x + 1) = 0$$

$$x = 3 \quad x = -1$$

(b)

$$A = \begin{bmatrix} 3 & 5 \\ 1 & 2 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 4 \\ 0 & -1 \end{bmatrix}$$

$$A = BCB^{-1}$$

$$B^{-1}(A) = B^{-1}(BCB^{-1})$$

$$B^{-1}(A) = ICB^{-1}$$

Singular Matrix

Singular matrix is square matrix whose determinant is zero.

$$B^{-1}A = CB^{-1}$$

$$(B^{-1}A)B = (CB^{-1})B$$

$$B^{-1}AB = CI$$

$$C = B^{-1}AB$$

$$A = \begin{bmatrix} 3 & 5 \\ 1 & 2 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 4 \\ 0 & -1 \end{bmatrix}$$

$$B^{-1} = \frac{1}{-1-0} \begin{bmatrix} -1 & -4 \\ 0 & 1 \end{bmatrix}$$

$$= -1 \begin{bmatrix} -1 & -4 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 4 \\ 0 & -1 \end{bmatrix}$$

$$C = B^{-1}AB$$

$$= \begin{bmatrix} 1 & 4 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 3 & 5 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 1 & 4 \\ 0 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 7 & 13 \\ -1 & -2 \end{bmatrix} \begin{bmatrix} 1 & 4 \\ 0 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 7 & 15 \\ -1 & -2 \end{bmatrix}$$

5. (a) Expand $(2 + x)^{-\frac{1}{2}}$ in ascending powers of x , up to the term x^3 .

(b) Use the expansion in (a) to approximate $\sqrt{\frac{2}{3}}$.

SOLUTION

(a)

$$\begin{aligned}
 (2 + x)^{-\frac{1}{2}} &= \left[2 \left(1 + \frac{x}{2}\right)\right]^{-\frac{1}{2}} \\
 &= 2^{-\frac{1}{2}} \left(1 + \frac{x}{2}\right)^{-\frac{1}{2}} \\
 &= \frac{1}{\sqrt{2}} \left[1 + \frac{\left(-\frac{1}{2}\right)}{1!} \left(\frac{x}{2}\right)^1 + \frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{2!} \left(\frac{x}{2}\right)^2 + \frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)\left(-\frac{5}{2}\right)}{3!} \left(\frac{x}{2}\right)^3 + \dots\right] \\
 &= \frac{1}{\sqrt{2}} \left[1 - \frac{x}{4} + \frac{3}{8} \left(\frac{x^2}{4}\right) - \frac{15}{48} \left(\frac{x^3}{8}\right) + \dots\right] \\
 &= \frac{1}{\sqrt{2}} \left[1 - \frac{x}{4} + \frac{3x^2}{32} - \frac{5x^3}{128} + \dots\right] \\
 &= \frac{1}{\sqrt{2}} - \frac{1}{4\sqrt{2}}x + \frac{3}{32\sqrt{2}}x^2 - \frac{5}{128\sqrt{2}}x^3 + \dots
 \end{aligned}$$

(b)

$$\begin{aligned}
 \sqrt{\frac{2}{3}} &= \left(\frac{2}{3}\right)^{\frac{1}{2}} \\
 &= \left(\frac{3}{2}\right)^{-\frac{1}{2}}
 \end{aligned}$$

$$(2+x)^{-\frac{1}{2}} = \left(\frac{3}{2}\right)^{-\frac{1}{2}}$$

$$2+x = \frac{3}{2}$$

$$x = -\frac{1}{2}$$

$$(2+x)^{-\frac{1}{2}} = \frac{1}{\sqrt{2}} - \frac{1}{4\sqrt{2}}x + \frac{3}{32\sqrt{2}}x^2 - \frac{5}{128\sqrt{2}}x^3$$

$$\left[2 + \left(-\frac{1}{2}\right)\right]^{-\frac{1}{2}} = \frac{1}{\sqrt{2}} - \frac{1}{4\sqrt{2}}\left(-\frac{1}{2}\right) + \frac{3}{32\sqrt{2}}\left(-\frac{1}{2}\right)^2 - \frac{5}{128\sqrt{2}}\left(-\frac{1}{2}\right)^3$$

$$\left(\frac{3}{2}\right)^{-\frac{1}{2}} = 0.8155$$

$$\left(\frac{2}{3}\right)^{\frac{1}{2}} = 0.8155$$

$$\sqrt{\frac{2}{3}} = 0.8155$$

6. Given $z_1 = 3 - 3i$ and $z_2 = 3 + 2i$.

(a) Write \bar{z}_1 in polar form.

(b) Express $\frac{(\bar{z}_1 z_2)}{13} + \left[\frac{i^3}{-z_2} \right]$ in the form of $a + bi, c, b \in R$.

SOLUTION

(a)

$$z_1 = 3 - 3i \text{ and } z_2 = 3 + 2i$$

$$\bar{z}_1 = 3 + 3i$$

$$r = |\bar{z}_1| = \sqrt{3^2 + 3^2}$$

$$= \sqrt{18}$$

$$= 3\sqrt{2}$$

$$\text{Argument of } \bar{z}_1, \quad \theta = \tan^{-1} \left(\frac{b}{a} \right)$$

$$= \tan^{-1} \left(\frac{3}{3} \right)$$

$$= \tan^{-1}(1)$$

$$= 0.785 \text{ rad}$$

\bar{z}_1 in polar form

$$\bar{z}_1 = r[\cos \theta + i \sin \theta]$$

$$= 3\sqrt{2}[\cos 0.785 + i \sin 0.785]$$

(b)

$$\begin{aligned}\frac{(\bar{z}_1 z_2)}{13} + \left(\frac{i^3}{-z_2}\right) &= \frac{(3+3i)(3+2i)}{13} + \left[\frac{-i}{-(3+2i)}\right] \\ &= \frac{9+6i+9i+6i^2}{13} + \left[\frac{i}{(3+2i)}\right] \\ &= \frac{3+15i}{13} + \left[\frac{i(3-2i)}{(3+2i)(3-2i)}\right] \\ &= \frac{3+15i}{13} + \left[\frac{3i+2}{9+4}\right] \\ &= \frac{3+15i}{13} + \left[\frac{2+3i}{13}\right] \\ &= \frac{3+15i}{13} + \frac{2-3i}{13} \\ &= \frac{3+15i+2-3i}{13} \\ &= \frac{5+12i}{13} \\ &= \frac{5}{13} + \frac{12}{13}i\end{aligned}$$

7. A curve $y = ax^2 + bx + c$ where a , b and c are constants, passes through the points $(2, 11)$, $(-1, -16)$ and $(3, 28)$.
- (a) By using the above information, construct a system containing three linear equations.
- (b) Express the above system as a matrix equation $AX=B$.
- (c) Find the inverse of matrix A by using the adjoint matrix method. Hence, obtain the values of a , b and c .

SOLUTION**(a)**

$$y = ax^2 + bx + c$$

$$\text{At } (2, 11) \rightarrow x = 2, y = 11$$

$$11 = a(2)^2 + b(2) + c$$

$$4a + 2b + c = 11 \quad \dots\dots\dots (1)$$

$$\text{At } (-1, -16) \rightarrow x = -1, y = -16$$

$$-16 = a(-1)^2 + b(-1) + c$$

$$a - b + c = -16 \quad \dots\dots\dots (2)$$

$$\text{At } (3, 28) \rightarrow x = 3, y = 28$$

$$28 = a(3)^2 + b(3) + c$$

$$9a + 3b + c = 28 \quad \dots\dots\dots (2)$$

(b)

$$4a + 2b + c = 11 \quad \dots\dots\dots (1)$$

$$a - b + c = -16 \quad \dots\dots\dots (2)$$

$$9a + 3b + c = 28 \quad \dots\dots\dots (2)$$

$$\begin{pmatrix} 4 & 2 & 1 \\ 1 & -1 & 1 \\ 9 & 3 & 1 \end{pmatrix} \begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} 11 \\ -16 \\ 28 \end{pmatrix}$$

(c)

$$A = \begin{pmatrix} 4 & 2 & 1 \\ 1 & -1 & 1 \\ 9 & 3 & 1 \end{pmatrix}$$

$$A^{-1} = \frac{1}{|A|} \text{Adj}(A)$$

$$\text{Adj}(A) = (C)^T$$

$$|A| = (4) \begin{vmatrix} -1 & 1 \\ 3 & 1 \end{vmatrix} - (2) \begin{vmatrix} 1 & 1 \\ 9 & 1 \end{vmatrix} + (1) \begin{vmatrix} 1 & -1 \\ 9 & 3 \end{vmatrix}$$

$$= 4[-1 - 3] - 2[1 - 9] + 1[3 + 9]$$

$$= -16 + 16 + 12$$

$$= 12$$

Cofactor of A

$$C = \begin{bmatrix} + \begin{vmatrix} -1 & 1 \\ 3 & 1 \end{vmatrix} & - \begin{vmatrix} 1 & 1 \\ 9 & 1 \end{vmatrix} & + \begin{vmatrix} 1 & -1 \\ 9 & 3 \end{vmatrix} \\ - \begin{vmatrix} 2 & 1 \\ 3 & 1 \end{vmatrix} & + \begin{vmatrix} 4 & 1 \\ 9 & 1 \end{vmatrix} & - \begin{vmatrix} 4 & 2 \\ 9 & 3 \end{vmatrix} \\ + \begin{vmatrix} 2 & 1 \\ -1 & 1 \end{vmatrix} & - \begin{vmatrix} 4 & 1 \\ 1 & 1 \end{vmatrix} & + \begin{vmatrix} 4 & 2 \\ 1 & -1 \end{vmatrix} \end{bmatrix}$$

$$= \begin{bmatrix} -1 - 3 & -(1 - 9) & 3 + 9 \\ -(2 - 3) & 4 - 9 & -(12 - 18) \\ 2 + 1 & -(4 - 1) & -4 - 2 \end{bmatrix}$$

$$= \begin{bmatrix} -4 & 8 & 12 \\ 1 & -5 & 6 \\ 3 & -3 & -6 \end{bmatrix}$$

Adjoin of A

$$\text{Adj}(A) = C^T$$

$$= \begin{bmatrix} -4 & 8 & 12 \\ 1 & -5 & 6 \\ 3 & -3 & -6 \end{bmatrix}^T$$

$$= \begin{bmatrix} -4 & 1 & 3 \\ 8 & -5 & -3 \\ 12 & 6 & -6 \end{bmatrix}$$

Inverse of A

$$A^{-1} = \frac{1}{|A|} \text{Adj}(A)$$

$$A^{-1} = \frac{1}{12} \begin{bmatrix} -4 & 1 & 3 \\ 8 & -5 & -3 \\ 12 & 6 & -6 \end{bmatrix}$$

$$= \begin{bmatrix} -\frac{4}{12} & \frac{1}{12} & \frac{3}{12} \\ \frac{8}{12} & -\frac{5}{12} & -\frac{3}{12} \\ \frac{12}{12} & \frac{6}{12} & -\frac{6}{12} \end{bmatrix}$$

$$= \begin{bmatrix} -\frac{1}{3} & \frac{1}{12} & \frac{1}{4} \\ \frac{2}{3} & -\frac{5}{12} & -\frac{1}{4} \\ 1 & \frac{1}{2} & -\frac{1}{2} \end{bmatrix}$$

$$\begin{pmatrix} 4 & 2 & 1 \\ 1 & -1 & 1 \\ 9 & 3 & 1 \end{pmatrix} \begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} 11 \\ -16 \\ 28 \end{pmatrix}$$

$$X = A^{-1}B$$

$$\begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} -\frac{1}{3} & \frac{1}{12} & \frac{1}{4} \\ \frac{2}{3} & -\frac{5}{12} & -\frac{1}{4} \\ 1 & \frac{1}{2} & -\frac{1}{2} \end{bmatrix} \begin{bmatrix} 11 \\ -16 \\ 28 \end{bmatrix}$$

$$= \begin{bmatrix} 2 \\ 7 \\ -11 \end{bmatrix}$$

$$\therefore a = 2, \quad b = 7, \quad c = -11$$

8. Given a function $f(x) = \sqrt{3 - 2x}$.
- (a) Show that f is a one to one function.
 - (b) Find the domain and range of f .
 - (c) Determine the inverse function of f and state its domain and range.
 - (d) Sketch the graphs of f and f^{-1} on the same axis.

SOLUTION

$$f(x) = \sqrt{3 - 2x}$$

(a)

$$f(x_1) = \sqrt{3 - 2x_1}$$

$$f(x_2) = \sqrt{3 - 2x_2}$$

$$f(x_1) = f(x_2)$$

$$\sqrt{3 - 2x_1} = \sqrt{3 - 2x_2}$$

$$3 - 2x_1 = 3 - 2x_2$$

$$x_1 = x_2$$

$\therefore f$ is a one to one function

(b)

$$f(x) = \sqrt{3 - 2x}$$

Domain of x :

$$D_f: \quad 3 - 2x \geq 0$$

$$3 \geq 2x$$

$$2x \leq 3$$

$$x \leq \frac{3}{2}$$

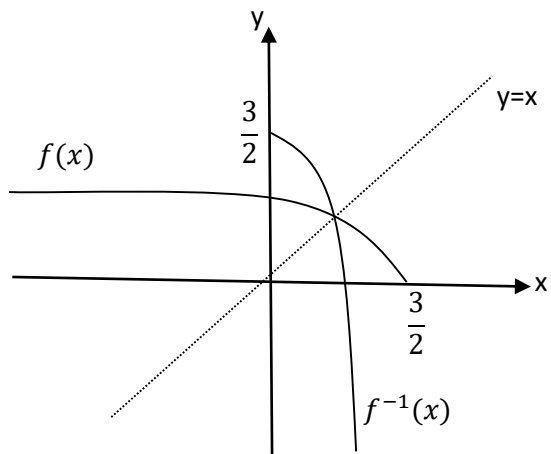
$$D_f = (-\infty, \frac{3}{2}]$$

$$R_f = [0, \infty)$$

(c)

	<u>Alternative</u>
$f(x) = \sqrt{3 - 2x}$	$f(x) = \sqrt{3 - 2x}$
$y = \sqrt{3 - 2x}$	$f[f^{-1}(x)] = x$
$y^2 = 3 - 2x$	$\sqrt{3 - 2f^{-1}(x)} = x$
$2x = 3 - y^2$	$3 - 2f^{-1}(x) = x^2$
$x = \frac{3 - y^2}{2}$	$2f^{-1}(x) = 3 - x^2$
$f^{-1}(x) = \frac{3 - x^2}{2}$	$f^{-1}(x) = \frac{3 - x^2}{2}$

(d)



9. (a) The function f is given as $f(x) = \frac{ax+2}{3x-4}, x \neq \frac{4}{3}$. If $(f \circ f)(x) = x$, find the value of a .

(b) Let $f(x) = \ln|3x + 2|$ and $g(x) = e^{-x} + 2$ be two functions. Evaluate $(g \circ f)^{-1}(3)$.

SOLUTION

$$f(x) = \frac{ax + 2}{3x - 4}, x \neq \frac{4}{3}$$

$$(f \circ f)(x) = x$$

$$f[f(x)] = x$$

$$f\left[\frac{ax + 2}{3x - 4}\right] = x$$

$$\frac{a\left[\frac{ax + 2}{3x - 4}\right] + 2}{3\left[\frac{ax + 2}{3x - 4}\right] - 4} = x$$

$$a\left[\frac{ax + 2}{3x - 4}\right] + 2 = x\left[3\left[\frac{ax + 2}{3x - 4}\right] - 4\right]$$

$$\left[\frac{a^2x + 2a}{3x - 4}\right] + \left[\frac{2(3x - 4)}{3x - 4}\right] = \left[\frac{3ax^2 + 6x}{3x - 4}\right] - 4x$$

$$\left[\frac{(a^2x + 2a) + 2(3x - 4)}{3x - 4}\right] = \left[\frac{3ax^2 + 6x}{3x - 4}\right] - \left[\frac{4x(3x - 4)}{3x - 4}\right]$$

$$\left[\frac{a^2x + 2a + 6x - 8}{3x - 4}\right] = \left[\frac{3ax^2 + 6x - 12x^2 + 16x}{3x - 4}\right]$$

$$a^2x + 2a + 6x - 8 = 3ax^2 + 6x - 12x^2 + 16x$$

$$3ax^2 - 12x^2 + 6x + 16x - a^2x - 6x - 2a + 8 = 0$$

$$(3a - 12)x^2 + (16 - a^2)x + 8 - 2a = 0$$

Compare coefficient of x^2

$$3a - 12 = 0$$

$$a = 4$$

(b)

$$f(x) = \ln|3x + 2|$$

$$g(x) = e^{-x} + 2$$

$$\begin{aligned}g \circ f(x) &= g[f(x)] \\&= g[\ln|3x + 2|] \\&= e^{-\ln|3x+2|} + 2 \\&= e^{\ln|3x+2|^{-1}} + 2 \\&= |3x + 2|^{-1} + 2 \\&= \frac{1}{3x + 2} + 2 \\&= \frac{1 + 2(3x + 2)}{3x + 2} \\&= \frac{1 + 6x + 4}{3x + 2} \\&= \frac{6x + 5}{3x + 2}\end{aligned}$$

$$\text{Let } y = \frac{6x+5}{3x+2}$$

$$y(3x + 2) = 6x + 5$$

$$3xy + 2y = 6x + 5$$

$$3xy - 6x = 5 - 2y$$

$$x(3y - 6) = 5 - 2y$$

$$x = \frac{5 - 2y}{3y - 6}$$

$$(g \circ f)^{-1}(y) = \frac{5 - 2y}{3y - 6}$$

$$(g \circ f)^{-1}(x) = \frac{5 - 2x}{3x - 6}$$

$$(g \circ f)^{-1}(3) = \frac{5 - 2(3)}{3(3) - 6} = -\frac{1}{3}$$

10. (a) Solve the inequality $\left| \frac{x-1}{x+3} \right| > 2$.

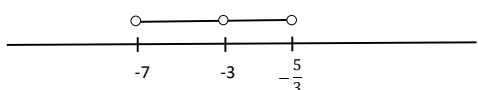
(b) Show that $\frac{2^x \times 4^{2x}}{8^x} = 2^{2x}$.

Hence, find the interval for x so that $\frac{2^x \times 4^{2x}}{8^x} - 13(2^x) + 36 \geq 0$.

SOLUTION

(a)

$$\left| \frac{x-1}{x+3} \right| > 2$$

$\frac{x-1}{x+3} > 2$ $\frac{x-1}{x+3} - 2 > 0$ $\frac{x-1}{x+3} - \frac{2(x+3)}{x+3} > 0$ $\frac{x-1-2x-6}{x+3} > 0$ $\frac{-x-7}{x+3} > 0$ <p>Let $-x-7=0 \rightarrow x = -7$</p> <p>$x+3=0 \rightarrow x = -3$</p> <table border="1" style="width: 100%; border-collapse: collapse; text-align: center;"> <tr> <td></td> <td>$(-\infty, -7)$</td> <td>$(-7, -3)$</td> <td>$(-3, \infty)$</td> </tr> <tr> <td>$-x-7$</td> <td>+</td> <td>-</td> <td>-</td> </tr> <tr> <td>$x+3$</td> <td>-</td> <td>-</td> <td>+</td> </tr> <tr> <td></td> <td>-</td> <td style="background-color: #cccccc;">+</td> <td>-</td> </tr> </table> <p style="text-align: center; color: green; font-weight: bold;">$(-7, -3)$</p>		$(-\infty, -7)$	$(-7, -3)$	$(-3, \infty)$	$-x-7$	+	-	-	$x+3$	-	-	+		-	+	-	Or	$\frac{x-1}{x+3} < -2$ $\frac{x-1}{x+3} + 2 < 0$ $\frac{x-1}{x+3} + \frac{2(x+3)}{x+3} < 0$ $\frac{x-1+2(x+3)}{x+3} < 0$ $\frac{x-1+2x+6}{x+3} < 0$ $\frac{3x+5}{x+3} < 0$ <p>Let $3x+5=0 \rightarrow x = -\frac{5}{3}$</p> <p>$x+3=0 \rightarrow x = -3$</p> <table border="1" style="width: 100%; border-collapse: collapse; text-align: center;"> <tr> <td></td> <td>$(-\infty, -3)$</td> <td>$(-3, -\frac{5}{3})$</td> <td>$(-\frac{5}{3}, \infty)$</td> </tr> <tr> <td>$3x+5$</td> <td>-</td> <td>-</td> <td>+</td> </tr> <tr> <td>$x+3$</td> <td>-</td> <td>+</td> <td>+</td> </tr> <tr> <td></td> <td>+</td> <td style="background-color: #cccccc;">-</td> <td>+</td> </tr> </table> <p style="text-align: center; color: green; font-weight: bold;">$(-3, -\frac{5}{3})$</p>		$(-\infty, -3)$	$(-3, -\frac{5}{3})$	$(-\frac{5}{3}, \infty)$	$3x+5$	-	-	+	$x+3$	-	+	+		+	-	+
	$(-\infty, -7)$	$(-7, -3)$	$(-3, \infty)$																															
$-x-7$	+	-	-																															
$x+3$	-	-	+																															
	-	+	-																															
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$3x+5$	-	-	+																															
$x+3$	-	+	+																															
	+	-	+																															
 <p style="text-align: center;">Solution Interval: $(-7, -3) \cup (-3, -\frac{5}{3})$</p>																																		

(b)

Show that $\frac{2^x \times 4^{2x}}{8^x} = 2^{2x}$

$$\frac{2^x \times 4^{2x}}{8^x} = \frac{2^x (2^2)^{2x}}{(2^3)^x}$$

$$= \frac{2^x 2^{4x}}{2^{3x}}$$

$$= \frac{2^{5x}}{2^{3x}}$$

$$= 2^{2x}$$

$$\frac{2^x \times 4^{2x}}{8^x} - 13(2^x) + 36 \geq 0$$

$$2^{2x} - 13(2^x) + 36 \geq 0$$

$$(2^x)^2 - 13(2^x) + 36 \geq 0$$

Let $u = 2^x$

$$(2^x)^2 - 13(2^x) + 36 \geq 0$$

$$u^2 - 13u + 36 \geq 0$$

$$(u - 9)(u - 4) \geq 0$$

$$\text{Let } (u - 9) = 0 \qquad (u - 4) = 0$$

$$u = 9$$

$$u = 4$$

	$(-\infty, 4)$	$(4, 9)$	$(9, \infty)$
$(u - 9)$	-	-	+
$(u - 4)$	-	+	+
$(u - 9)(u - 4)$	+	-	+

$$u \leq 4 \quad \text{or} \quad u \geq 9$$

$$2^x \leq 4 \quad \text{or} \quad 2^x \geq 9$$

$$\ln 2^x \leq \ln 4 \quad \text{or} \quad \ln 2^x \geq \ln 9$$

$$x \ln 2 \leq \ln 4 \quad \text{or} \quad x \ln 2 \geq \ln 9$$

$$x \leq \frac{\ln 4}{\ln 2} \quad \text{or} \quad x \geq \frac{\ln 9}{\ln 2}$$

$$x \leq 2 \quad \text{or} \quad x \geq 3.170$$

$$\text{Solution interval : } (-\infty, 2] \cup [3.17, \infty)$$