QS026/2
Mathematics
Paper 2
Semester II
Session 2005/2006
2 hours
QS026/2
Matematik
Kertas 2


Semester II Sesi 2005/2006

2 jam

# BAHAGIAN MATRIKULASI <br> KEMENTERIAN PELAJARAN MALAYSIA <br> MATRICULATION DIVISION <br> MINISTRY OF EDUCATION MALAYSIA 

## PEPERIKSAAN SEMESTER PROGRAM MATRIKULASI

MATRICULATION PROGRAMME EXAMINATION

## MATEMATIK

## Kertas 2

2 jam

## JANGAN BUKA KERTAS SOALAN INI SEHINGGA DIBERITAHU.

DO NOT OPEN THIS BOOKLET UNTIL YOU ARE TOLD TO DO SO.

Kertas soalan ini mengandungi 11 halaman bercetak.
This booklet consists of 11 printed pages.

## INSTRUCTIONS TO CANDIDATE:

This question booklet consists of $\mathbf{1 0}$ questions.
Answer all questions.
The full marks for each question or section are shown in the bracket at the end of each of the question or section.

All steps must be shown clearly.
Only non-programmable scientific calculators can be used.
Numerical answers can be given in the form of $\pi$, e, surd, fractions or up to three significant figures, where appropriate, unless stated otherwise in the question.

1. The cumulative distribution for the ages of 50 patients at a clinic on a particular day is shown in the table below.

| Age (Years) | Number of Patients |
| :---: | :---: |
| $<2$ | 0 |
| $<12$ | 5 |
| $<22$ | 12 |
| $<32$ | 29 |
| $<42$ | 38 |
| $<52$ | 45 |
| $<62$ | 50 |

Calculate the
(a) median.
(b) mode.
2. How many ways can all the alphabets from the word PUTRAJAYA be arranged if
(a) the first letter is $\mathbf{P}$ and the last letter is $\mathbf{A}$ ?
(b) all the letters A must be next to each other?
(c) they begin with a consonant and end with a vowel?
3. In a cooking competition, 6 out of 10 contestants are females. In how many ways can one choose
(a) the champion, first and second runners-up?
(b) four winners consisting of at least two females?
(c) winners for first, second and third place consisting of two females and a male?
[2 marks]
4. Given $A$ and $B$ are two events with the following probabilities:

$$
\mathrm{P}(\mathrm{~A})=\frac{2}{7}, \mathrm{P}\left(\mathrm{~A} \mid \mathrm{B}^{\prime}\right)=\frac{3}{7} \text { and } \mathrm{P}(\mathrm{~A} \cap \mathrm{~B})=\frac{1}{21} .
$$

(a) Find $\mathrm{P}(\mathrm{A} \cup \mathrm{B})$.
(b) Determine whether A and B are independent events.
5. Consider a sample consisting of the following observations:

$$
\begin{array}{llllllll}
24 & 12 & 22 & 12 & 20 & 32 & 4 & 10
\end{array}
$$

(a) Calculate the mean, median and mode.
(b) A new sample is formed using all the above observations with the three values in part (a) included. For this new sample, calculate the
(i) first quartile.
(ii) standard deviation.
6. A game involves rolling a fair die followed by drawing a marble from either an urn A or B. If the outcome is less than 3 , then a marble is drawn at random from urn A . Otherwise, a marble is drawn at random from urn B. Urn A contains 3 red marbles, 4 blue marbles, and 3 green marbles. Urn B contains 3 red marbles and 1 blue marble.
(a) Find the probability that
(i) a red marble is chosen.
(ii) the outcome of the die is less than 3 if it is known that the marble drawn is red.
(iii) it is a red or a green marble.
(b) The similar game is repeated but two marbles are drawn instead from either urn A or B. Find the probability that both marbles are red if the first marble is taken
(i) with replacement.
(ii) without replacement.
7. The probability distribution of a discrete random variable $X$ is given as follows:

| $X$ | -1 | 0 | 1 | 2 | 3 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{P}(X=x)$ | $p$ | $3 p$ | $2 p$ | $p$ | $p$ |

Show that $p=\frac{1}{8}$.
[2 marks]
[2 marks]
(b) Find
(i) the mean.
(ii) the median.
(iii) $\mathrm{E}\left((3 X-1)^{2}\right)$.
8. The continuous random variable $X$ has the probability density function of

$$
f(x)= \begin{cases}a \sqrt{|x|+x} & ,-1 \leq x \leq 1 \\ 0 & , \text { otherwise }\end{cases}
$$

Show that $a=\frac{3 \sqrt{2}}{4}$.
(a) Find $\mathrm{P}\left(0<X<\frac{1}{2}\right)$ and give your answer in the surd form. [2 marks]
(b) Find the median and the expected value of $X$.
9. The number of short messages (SMS) received by a teenager in half an hour has a Poisson distribution with mean $\mu$.
(a) If the probability of receiving no SMS within half an hour is 0.0025 , show that $\mu=6$ (to the nearest integer).
(b) Using the value of $\mu=6$, find the probability that
(i) he receives less than six SMS in half an hour.
(ii) he receives less than six SMS in one hour.
(iii) two teenagers selected at random will receive at least six SMS in half an hour.
10. In any of its shipments, a company found that the probability of bad oranges it supplies is 0.2 . At the receiving terminal, a sample is taken at random and the number of bad oranges is recorded.
(a) A shipment will be rejected if there are more than $10 \%$ bad oranges in the sample taken. Calculate the probability that a particular shipment will be accepted if a sample of size 20 is taken.
(b) Using the normal approximation, estimate the probability of obtaining 180 to 210 bad oranges if 1000 oranges are inspected at random.
(c) In another shipment, the probability of obtaining bad oranges is 0.03 . The probability of rejecting this shipment is 0.022 . Using the Poisson approximation, determine the maximum allowable number of bad oranges in a sample of size 300 such that the shipment is accepted.

