QS025/2
Matematik
Kertas 2
Semester II
Sesi 2013/2014
2 jam

KEMENTERIAN PENDIDIKAN MALAYSIA

BAHAGIAN MATRIKULASI
MATRICULATION DIVISION

PEPERIKSAAN SEMESTER PROGRAM MATRIKULASI
MATRICULATION PROGRAMME EXAMINATION

MATEMATIK
Kertas 2
2 jam

JANGAN BUKA KERTAS SOALAN INI SEHINGGA DIBERITAHU.
DO NOT OPEN THIS QUESTION PAPER UNTIL YOU ARE TOLD TO DO SO.

Kertas soalan ini mengandungi **19** halaman bercetak.
This question paper consists of **19** *printed pages.*

INSTRUCTIONS TO CANDIDATE:

This question paper consists of **10** questions.

Answer **all** questions.

All answers must be written in the answer booklet provided. Use a new page for each question.

The full marks for each question or section are shown in the bracket at the end of the question or section.

All steps must be shown clearly.

Only non-programmable scientific calculators can be used.

Numerical answers may be given in the form of π, e, surd, fractions or up to three significant figures, where appropriate, unless stated otherwise in the question.

LIST OF MATHEMATICAL FORMULAE

Statistics

For ungrouped data, the *k*th percentile,

$$P_k = \begin{cases} \dfrac{x_{(s)} + x_{(s+1)}}{2}, & \text{if } s \text{ is an integer} \\ x_{([s])}, & \text{if } s \text{ is a non-integer} \end{cases}$$

where $s = \dfrac{n \times k}{100}$ and $[\,s\,]$ = the least integer greater than k.

For grouped data, the *k*th percentiles, $P_k = L_k + \left[\dfrac{\left(\dfrac{k}{100}\right)n - F_{k-1}}{f_k} \right] c.$

For grouped data, the mode, $M = L_M + \left[\dfrac{d_1}{d_1 + d_2} \right] c.$

Variance

$$s^2 = \frac{\sum f_i x_i^2 - \frac{1}{n}\left(\sum f_i x_i\right)^2}{n-1}$$

Binomial Distribution

$$X \sim B(n, p)$$

$$P(X = x) = {}^nC_x p^x (1-p)^{n-x}, \quad x = 0, 1, 2, 3, \ldots, n$$

Poisson Distribution

$$X \sim P_o(\lambda)$$

$$P(X = x) = \frac{e^{-\lambda}\lambda^x}{x!}, \quad x = 0, 1, 2, 3, \ldots$$

1 The weights (in kg) to the nearest integer of ten lecturers in a college are given as follows:

$$51, 74, 59, 59, w, 51, 60, 70, w+8, 56$$

where w is an integer. If the mean weight is 65 kg, determine the value of w. Hence, find the mode and the 80^{th} percentile.

[6 *marks*]

2 The events A and B are independent with $P(A) = x$, $P(B) = x + 0.3$ and $P(A \cap B) = 0.04$. Determine the value of x. Hence, find $P(A \cup B)'$.

[6 *marks*]

3 A discrete random variable X has a Poisson distribution with parameter λ. By using its probability distribution function, show that $P(X = y+1) = \dfrac{\lambda}{y+1} P(X = y)$ where y is an integer. Given that $E(X) = 1.5$, find $P(X = 2)$.

[6 *marks*]

4 The probability distribution function of a discrete random variable X is given as follows:

X	1	2	3	4	5	6
$P(X = x)$	$\frac{1}{k}$	$\frac{1}{k}$	$\frac{3}{k}$	$\frac{2}{k}$	$\frac{5}{k}$	$\frac{3}{k}$

where k is a constant. Determine the value of k. Hence, calculate $\text{Var}\,(2X - 1)$.

[7 *marks*]

5 The following table shows the frequency distribution of the number of children at a childcare center according to their ages (in years).

Age	Frequency
1.0 – 2.0	12
2.0 – 3.0	8
3.0 – 4.0	2
4.0 – 5.0	4
5.0 – 6.0	10

(a) Calculate the mean, mode and median. Hence, state the shape of the distribution of the data and give your reason.

[8 *marks*]

(b) Calculate and interpret the value of the third quartile.

[3 *marks*]

6 (a) In the final of a science quiz competition, teams A and B sit in rows facing each other. Each team consists of two females and two males. Find the number of different seating arrangements of all the contestants if

(i) in each team, contestants of the same gender request to sit next to each other.

[3 *marks*]

(ii) contestants of the same gender do not sit next to each other in team A, or in team B, contestants of the same gender sit at both ends of the row.

[3 *marks*]

(b) A test consists of ten true-false questions. How many possible arrangements of answers can be obtained if

(i) all questions must be answered?

[2 *marks*]

(ii) only six randomly chosen questions must be answered?

[4 *marks*]

7 The number of vehicles owned by residents in a housing estate, X is a discrete random variable with probability distribution function

$$f(x)=\begin{cases}\dfrac{1}{120}, & x=0\\[2ex] \dfrac{x}{(x+1)!}, & x=1,\ 2,\ 3,\ 4.\end{cases}$$

(a) Verify that $f(x)$ is a probability distribution function.

[2 *marks*]

(b) Find the probability that a resident has more than two vehicles.

[2 *marks*]

(c) Find the cumulative distribution function for X and hence determine the median.

[5 *marks*]

(d) Let $Y = 30X + 10$ be the monthly fee (in RM) imposed by a security company. Find the expected amount of monthly fee to be paid.

[3 *marks*]

8 The probability that a person is a carrier of Thalassemia is 0.03. If a person is actually a carrier, the probability a medical diagnostic test will give a positive result, indicating that he is a carrier, is 0.92. If the person is actually not a carrier, the probability of a positive result is 0.03. Draw a tree diagram to represent the given information.

[2 *marks*]

(a) A medical diagnostic test is said to be efficient if 5% of the time it gives a correct positive result. Determine if the test is efficient.

[3 *marks*]

(b) Find the probability that the test gives a negative result.

[2 *marks*]

(c) What is the probability that the diagnostic test gives a negative result and the person tested is not a carrier?

[2 *marks*]

(d) Two persons who took the test are randomly chosen. What is the probability that both give positive results?

[3 *marks*]

9 The time for patients to experience complications in a week after a heart surgery, X is a continuous random variable with probability density function given by

$$f(x)=\begin{cases} a(1-x), & 0\le x\le 1 \\ 0, & \text{otherwise.} \end{cases}$$

where a is a constant. Show that $a=2$.

[2 *marks*]

Hence, find the

(a) cumulative distribution function of X and estimate its median.

[4 *marks*]

(b) mean and variance of X. Calculate $\text{Var}(3-2X)$.

[7 *marks*]

10 The lifetime of D sized batteries produced by a local factory is normally distributed with mean 11.5 months and standard deviation 0.8 months.

(a) Suppose a battery is selected at random from the factory's production line.

(i) Calculate the probability that the battery's lifetime is between 9.5 and 11.5 months, correct to one decimal place.

[3 *marks*]

(ii) If the probability that the battery's lifetime is less than h months is 0.975, determine the value of h.

[4 *marks*]

(b) Suppose ten batteries are selected at random from the factory's production line, calculate the probability that at most three batteries have lifetime between 9.5 and 11.5 months.

[3 *marks*]

(c) If 100 batteries are selected at random from the factory's production line, approximate the probability that from 48 to 51 batteries have lifetime between 9.5 and 11.5 months.

[5 *marks*]

END OF QUESTION PAPER