SULIT

QS025/1
Mathematics
Paper 1
Semester II
Session 2015/2016
2 hours

QS025/1
Matematik
Kertas 1
Semester II
Sesi 2015/2016
2 jam

KEMENTERIAN
PENDIDIKAN
MALAYSIA

BAHAGIAN MATRIKULASI
MATRICULATION DIVISION

PEPERIKSAAN SEMESTER PROGRAM MATRIKULASI
MATRICULATION PROGRAMME EXAMINATION

MATEMATIK
Kertas 1
2 jam

JANGAN BUKA KERTAS SOALAN INI SEHINGGA DIBERITAHU.
DO NOT OPEN THIS QUESTION PAPER UNTIL YOU ARE TOLD TO DO SO.

CHOW CHOON WOOI

ARAHAN KEPADA CALON:

Kertas soalan ini mengandungi **10** soalan.

Jawab **semua** soalan.

Semua jawapan hendaklah ditulis pada buku jawapan yang disediakan. Gunakan muka surat baru bagi nombor soalan yang berbeza.

Markah penuh yang diperuntukkan bagi setiap soalan atau bahagian soalan ditunjukkan dalam kurungan pada penghujung soalan atau bahagian soalan.

Semua langkah kerja hendaklah ditunjukkan dengan jelas.

Kalkulator saintifik yang tidak boleh diprogramkan sahaja yang boleh digunakan.

Jawapan berangka boleh diberi dalam bentuk π, e, surd, pecahan atau sehingga tiga angka bererti, di mana-mana yang sesuai, kecuali jika dinyatakan dalam soalan.

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INSTRUCTIONS TO CANDIDATE:

This question paper consists of **10** questions.

Answer **all** questions.

All answers must be written in the answer booklet provided. Use a new page for each question.

The full marks for each question or section are shown in the bracket at the end of the question or section.

All steps must be shown clearly.

Only non-programmable scientific calculators can be used.

Numerical answers may be given in the form of π, e, surd, fractions or up to three significant figures, where appropriate, unless stated otherwise in the question.

SENARAI RUMUS MATEMATIK

Trigonometri

$$\sin (A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos (A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan (A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin A + \sin B = 2 \sin \frac{A+B}{2} \cos \frac{A-B}{2}$$

$$\sin A - \sin B = 2 \cos \frac{A+B}{2} \sin \frac{A-B}{2}$$

$$\cos A + \cos B = 2 \cos \frac{A+B}{2} \cos \frac{A-B}{2}$$

$$\cos A - \cos B = 2 \sin \frac{A+B}{2} \sin \frac{A-B}{2}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A$$
$$= 2 \cos^2 A - 1$$
$$= 1 - 2 \sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

$$\sin^2 A = \frac{1 - \cos 2A}{2}$$

$$\cos^2 A = \frac{1 + \cos 2A}{2}$$

$$\cos^2 x + \sin^2 x = 1$$

$$1 + \tan^2 x = \sec^2 x$$

$$\cot^2 x + 1 = \operatorname{cosec}^2 x$$

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LIST OF MATHEMATICAL FORMULAE

Trigonometry

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin A + \sin B = 2 \sin \frac{A+B}{2} \cos \frac{A-B}{2}$$

$$\sin A - \sin B = 2 \cos \frac{A+B}{2} \sin \frac{A-B}{2}$$

$$\cos A + \cos B = 2 \cos \frac{A+B}{2} \cos \frac{A-B}{2}$$

$$\cos A - \cos B = 2 \sin \frac{A+B}{2} \sin \frac{A-B}{2}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A$$
$$= 2 \cos^2 A - 1$$
$$= 1 - 2 \sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

$$\sin^2 A = \frac{1 - \cos 2A}{2}$$

$$\cos^2 A = \frac{1 + \cos 2A}{2}$$

$$\cos^2 x + \sin^2 x = 1$$

$$1 + \tan^2 x = \sec^2 x$$

$$\cot^2 x + 1 = \operatorname{cosec}^2 x$$

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SENARAI RUMUS MATEMATIK

Pembezaan dan Pengamiran

$$\frac{d}{dx}(\sin x) = \cos x$$

$$\frac{d}{dx}(\cos x) = -\sin x$$

$$\frac{d}{dx}(\tan x) = \sec^2 x$$

$$\frac{d}{dx}(\cot x) = -\csc^2 x$$

$$\frac{d}{dx}(\sec x) = \sec x \tan x$$

$$\frac{d}{dx}(\csc x) = -\csc x \cot x$$

$$\int f'(x)e^{f(x)}\,dx = e^{f(x)} + c$$

$$\int \frac{f'(x)}{f(x)}\,dx = \ln|f(x)| + c$$

$$\int f'(x)[f(x)]^n\,dx = \frac{[f(x)]^{n+1}}{n+1} + c, \ n \neq -1$$

$$\int u\,dv = uv - \int v\,du$$

Sfera	$V = \dfrac{4}{3}\pi r^3$	$S = 4\pi r^2$
Kon membulat tegak	$V = \dfrac{1}{3}\pi r^2 h$	$S = \pi r^2 + \pi rh$
Silinder membulat tegak	$V = \pi r^2 h$	$S = 2\pi r^2 + 2\pi rh$

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LIST OF MATHEMATICAL FORMULAE

Differentiation and Integration

$$\frac{d}{dx}(\sin x) = \cos x$$

$$\frac{d}{dx}(\cos x) = -\sin x$$

$$\frac{d}{dx}(\tan x) = \sec^2 x$$

$$\frac{d}{dx}(\cot x) = -\csc^2 x$$

$$\frac{d}{dx}(\sec x) = \sec x \tan x$$

$$\frac{d}{dx}(\csc x) = -\csc x \cot x$$

$$\int f'(x)e^{f(x)}\,dx = e^{f(x)} + c$$

$$\int \frac{f'(x)}{f(x)}\,dx = \ln|f(x)| + c$$

$$\int f'(x)[f(x)]^n\,dx = \frac{[f(x)]^{n+1}}{n+1} + c,\ n \neq -1$$

$$\int u\,dv = uv - \int v\,du$$

Sphere	$V = \dfrac{4}{3}\pi r^3$	$S = 4\pi r^2$
Right circular cone	$V = \dfrac{1}{3}\pi r^2 h$	$S = \pi r^2 + \pi r h$
Right circular cylinder	$V = \pi r^2 h$	$S = 2\pi r^2 + 2\pi r h$

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SENARAI RUMUS MATEMATIK

Kaedah Berangka

Kaedah Newton-Raphson:

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}, \quad n = 1, 2, 3, \ldots$$

Petua Trapezium:

$$\int_a^b f(x)\,dx \approx \frac{h}{2}[(y_0 + y_n) + 2(y_1 + y_2 + \ldots + y_{n-1})], \quad h = \frac{b-a}{n}$$

Keratan Kon

Bulatan:

$$(x-h)^2 + (y-k)^2 = r^2$$

$$x^2 + y^2 + 2gx + 2fy + c = 0$$

$$xx_1 + yy_1 + g(x+x_1) + f(y+y_1) + c = 0$$

$$r = \sqrt{f^2 + g^2 - c}$$

$$r = \frac{|ah + bk + c|}{\sqrt{a^2 + b^2}}$$

Parabola:

$$(x-h)^2 = 4p(y-k)$$

$$(y-k)^2 = 4p(x-h)$$

$$F(h+p, k) \text{ atau } F(h, k+p)$$

Elips:

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$

$$F(h\pm c, k) \text{ atau } F(h, k\pm c)$$

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LIST OF MATHEMATICAL FORMULAE

Numerical Methods

Newton-Raphson Method:

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}, \quad n = 1, 2, 3, \ldots.$$

Trapezoidal Rule:

$$\int_a^b f(x)\,dx \approx \frac{h}{2}[(y_0 + y_n) + 2(y_1 + y_2 + \ldots + y_{n-1})], \quad h = \frac{b-a}{n}$$

Conics

Circle:

$$(x-h)^2 + (y-k)^2 = r^2$$

$$x^2 + y^2 + 2gx + 2fy + c = 0$$

$$xx_1 + yy_1 + g(x + x_1) + f(y + y_1) + c = 0$$

$$r = \sqrt{f^2 + g^2 - c}$$

$$r = \frac{|ah + bk + c|}{\sqrt{a^2 + b^2}}$$

Parabola:

$$(x-h)^2 = 4p(y-k)$$

$$(y-k)^2 = 4p(x-h)$$

$$F(h+p, k) \text{ or } F(h, k+p)$$

Ellipse:

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$

$$F(h \pm c, k) \text{ or } F(h, k \pm c)$$

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1   Cari persamaan bagi suatu bulatan yang melalui titik $(1,2)$, $(-1,2)$ dan $(0,-1)$. Seterusnya, tentukan pusatnya.

<div align="right">[5 <i>markah</i>]</div>

2   Tunjukkan bahawa $\int_{1}^{e} x \ln x \, dx = \frac{1}{4}(1+e^2)$.

<div align="right">[6 <i>markah</i>]</div>

3   Cari y dalam sebutan x diberi bahawa

$$x\frac{dy}{dx} = (1-2x^2)y$$

dengan $x > 0$ dan $y = 1$ apabila $x = 1$.

<div align="right">[7 <i>markah</i>]</div>

4   Cari penyelesaian am bagi persamaan pembezaan

$$\frac{dy}{dx} + y\cot x = 2\sin x.$$

<div align="right">[7 <i>markah</i>]</div>

5   Ungkapkan $\dfrac{1-4x}{3+x-2x^2}$ dalam pecahan separa dan seterusnya, cari nilai tepat bagi

$$\int_{0}^{1} \frac{1-4x}{3+x-2x^2} \, dx.$$

<div align="right">[10 <i>markah</i>]</div>

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1   Find the equation of a circle that is passing through points $(1,2)$, $(-1,2)$ and

$(0,-1)$. Hence, determine its center.

[5 *marks*]

2   Show that $\int_1^e x \ln x\, dx = \frac{1}{4}(1+e^2)$.

[6 *marks*]

3   Find y in terms of x given that

$$x\frac{dy}{dx} = (1-2x^2)y$$

where $x > 0$ and $y = 1$ when $x = 1$.

[7 *marks*]

4   Find the general solution of the differential equation

$$\frac{dy}{dx} + y\cot x = 2\sin x.$$

[7 *marks*]

5   Express $\dfrac{1-4x}{3+x-2x^2}$ in partial fractions and hence, find the exact value of

$$\int_0^1 \frac{1-4x}{3+x-2x^2}\, dx.$$

[10 *marks*]

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6   (a)   Diberi $f_1(x) = 2x$ dan $f_2(x) = -\ln x$.

   (i)   Tanpa menggunakan lakaran graf, tunjukkan bahawa $y = f_1(x)$ dan $y = f_2(x)$ bersilang dalam selang $[0.1, 1]$.

   [2 markah]

   (ii)   Gunakan kaedah Newton-Raphson untuk menganggarkan titik persilangan $y = f_1(x)$ dan $y = f_2(x)$, dengan nilai awal $x_1 = 1$. Lakukan lelaran sehingga $|f(x_n)| < 0.005$. Berikan jawapan anda betul kepada tiga tempat perpuluhan.

   [5 markah]

   (b)   Dengan menggunakan petua trapezium, cari nilai hampiran bagi $\int_0^1 x\sqrt{x+1}\, dx$ apabila $n = 4$, betul kepada empat tempat perpuluhan.

   [5 markah]

7   (a)   Jika $p = 3\underset{\sim}{i} - \underset{\sim}{j} + 2\underset{\sim}{k}$ dan $q = 2\underset{\sim}{i} + 2\underset{\sim}{j} - \underset{\sim}{k}$, tunjukkan bahawa

   $$\left| \underset{\sim}{p} \times \underset{\sim}{q} \right|^2 = \left| \underset{\sim}{p} \right|^2 \left| \underset{\sim}{q} \right|^2 - \left(\underset{\sim}{p} \cdot \underset{\sim}{q} \right)^2.$$

   [7 markah]

   (b)   Diberi satu segitiga ABC dengan $\overrightarrow{AB} = 2\underset{\sim}{a}$ dan $\overrightarrow{AC} = 3\underset{\sim}{b}$. Gunakan keputusan dalam bahagian (a), tunjukkan bahawa luas segitiga tersebut adalah $3\sqrt{\left| \underset{\sim}{a} \right|^2 \left| \underset{\sim}{b} \right|^2 - \left(\underset{\sim}{a} \cdot \underset{\sim}{b} \right)^2}$.

   Seterusnya, deduksikan luas segitiga tersebut jika $\underset{\sim}{a} = \underset{\sim}{p}$ dan $\underset{\sim}{b} = \underset{\sim}{q}$.

   [5 markah]

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6   (a)   Given $f_1(x) = 2x$ and $f_2(x) = -\ln x$.

         (i)   Without using curve sketching, show that $y = f_1(x)$ and $y = f_2(x)$ intersect on the interval of $[0.1, 1]$.

                                                                         [2 *marks*]

         (ii)   Use Newton-Raphson's method to estimate the intersection point of $y = f_1(x)$ and $y = f_2(x)$, with the initial value $x_1 = 1$. Iterate until $|f(x_n)| < 0.005$. Give your answer correct to three decimal places.

                                                                         [5 *marks*]

    (b)   By using the trapezoidal rule, find the approximate value for $\int_0^1 x\sqrt{x+1}\ dx$ when $n = 4$, correct to four decimal places.

                                                                         [5 *marks*]

7   (a)   If $p = 3\underset{\sim}{i} - \underset{\sim}{j} + 2\underset{\sim}{k}$ and $q = 2\underset{\sim}{i} + 2\underset{\sim}{j} - \underset{\sim}{k}$, show that

$$\left| \underset{\sim}{p} \times \underset{\sim}{q} \right|^2 = \left| \underset{\sim}{p} \right|^2 \left| \underset{\sim}{q} \right|^2 - \left(\underset{\sim}{p} \cdot \underset{\sim}{q} \right)^2.$$

                                                                         [7 *marks*]

    (b)   Given a triangle ABC with $\overrightarrow{AB} = 2\underset{\sim}{a}$ and $\overrightarrow{AC} = 3\underset{\sim}{b}$. Use the result in part (a), show that the area of the triangle is $3\sqrt{\left| \underset{\sim}{a} \right|^2 \left| \underset{\sim}{b} \right|^2 - \left(\underset{\sim}{a} \cdot \underset{\sim}{b} \right)^2}$.

Hence, deduce the area of the triangle if $\underset{\sim}{a} = \underset{\sim}{p}$ and $\underset{\sim}{b} = \underset{\sim}{q}$.

                                                                         [5 *marks*]

8   Diberi garis $l: x = 2 - t, \quad y = -3 + 4t, \quad z = -5 - 3t$ dan dua satah

$\pi_1: 2x - y + 7z = 53$ dan $\pi_2: 3x + y + z = 1$. Cari

(a)   titik persilangan antara garis l dan satah π_1.

[3 *markah*]

(b)   sudut tirus antara garis l dan satah π_1.

[6 *markah*]

(c)   sudut tirus antara satah π_1 dan satah π_2.

[4 *markah*]

9   (a)   Cari persamaan dalam bentuk piawai bagi suatu elips yang melalui titik $(-1, 6)$ dan mempunyai fokus $(-5, 2)$ dan $(3, 2)$.

[10 *markah*]

(b)   Berdasarkan kepada keputusan dalam bahagian (a), lakarkan graf bagi elips tersebut.

[3 *markah*]

8   Given a line $l: x = 2-t, \quad y = -3+4t, \quad z = -5-3t$ and two planes

$\pi_1: 2x - y + 7z = 53$ and $\pi_2: 3x + y + z = 1$. Find

    (a)   the point of intersection between the line l and the plane π_1.

                                                             [3 *marks*]

    (b)   the acute angle between the line l and the plane π_1.

                                                             [6 *marks*]

    (c)   the acute angle between planes π_1 and π_2.

                                                             [4 *marks*]

9   (a)   Find the equation in standard form of an ellipse which passes through the point $(-1, 6)$ and having foci at $(-5, 2)$ and $(3, 2)$.

                                                           [10 *marks*]

    (b)   From the result obtained in part (a), sketch the graph of the ellips.

                                                           [3 *marks*]

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10   (a)   Lakar dan lorekkan rantau R yang dibatasi oleh lengkung $y = \sqrt{x}$, garis $y = 2 - x$ dan paksi-y. Seterusnya, cari luas rantau R.

[7 markah]

       (b)   Jika R_1 adalah rantau yang dibatasi oleh lengkung $y = \sqrt{x}$, garis $y = 2 - x$ dan paksi-x, deduksikan nisbah $R : R_1$.

[3 markah]

       (c)   Cari isipadu pepejal yang terjana apabila rantau R diputar $360°$ pada paksi-x.

[5 markah]

KERTAS SOALAN TAMAT

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10   (a)   Sketch and shade the region R bounded by the curve $y = \sqrt{x}$, line

$y = 2 - x$ and y-axis. Hence, find the area of the region R.

[7 *marks*]

(b)   If R_1 is a region bounded by the curve $y = \sqrt{x}$, line $y = 2 - x$ and x-axis,

deduce the ratio of $R : R_1$.

[3 *marks*]

(c)   Find the volume of the solid generated when the region R is rotated through

$360°$ about the x-axis.

[5 *marks*]

END OF QUESTION PAPER

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