QS026
Mathematics
Semester 2
2008/2009
1 hour

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Matematik
Semester II
2008/2009
1 jam

BAHAGIAN MATRIKULASI
KEMENTERIAN PELAJARAN MALAYSIA
MATRICULATION DIVISION
MINISTRY OF EDUCATION MALAYSIA

UJIAN PERTENGAHAN SEMESTER PROGRAM MATRIKULASI
MID-SEMESTER EXAMINATION

MATEMATIK
1 jam

JANGAN BUKA KERTAS SOALAN INI SEHINGGA DIBERITAHU.
DO NOT OPEN THIS BOOKLET UNTIL YOU ARE TOLD TO DO SO.

ARAHAN KEPADA CALON:

Kertas soalan ini mengandungi **7** soalan.

Jawab **semua** soalan.

Markah penuh yang diperuntukkan bagi tiap-tiap soalan atau bahagian soalan ditunjukkan dalam kurungan pada penghujung soalan atau bahagian soalan.

Semua langkah kerja hendaklah ditunjukkan dengan jelas.

Kalkulator saintifik yang tidak boleh diprogramkan sahaja boleh digunakan.

Jawapan berangka boleh diberi dalam bentuk π, e, surd, pecahan atau sehingga tiga angka bererti, di mana-mana yang sesuai, kecuali jika dinyatakan dalam soalan.

INSTRUCTIONS TO CANDIDATE:

This question booklet consists of **7** questions.

Answer **all** questions.

The full marks for each question or section are shown in the bracket at the end of each of the question or section.

All steps must be shown clearly.

Only non-programmable scientific calculators can be used.

Numerical answers can be given in the form of π, e, surd, fractions or up to three significant figures, where appropriate, unless stated otherwise in the question.

LIST OF MATHEMATICAL FORMULAE

Trigonometry

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin A + \sin B = 2 \sin\left(\frac{A+B}{2}\right) \cos\left(\frac{A-B}{2}\right)$$
$$\sin A - \sin B = 2 \cos\left(\frac{A+B}{2}\right) \sin\left(\frac{A-B}{2}\right)$$
$$\cos A + \cos B = 2 \cos\left(\frac{A+B}{2}\right) \cos\left(\frac{A-B}{2}\right)$$
$$\cos A - \cos B = -2 \sin\left(\frac{A+B}{2}\right) \sin\left(\frac{A-B}{2}\right)$$

Hyperbolic

$$\sinh(x \pm y) = \sinh x \cosh y \pm \cosh x \sinh y$$
$$\cosh(x \pm y) = \cosh x \cosh y \pm \sinh x \sinh y$$
$$\cosh^2 x - \sinh^2 x = 1$$
$$1 - \tanh^2 x = \operatorname{sech}^2 x$$
$$\coth^2 x - 1 = \operatorname{cosech}^2 x$$
$$\sinh 2x = 2 \sinh x \cosh x$$
$$\cosh 2x = \cosh^2 x + \sinh^2 x$$

Differentiation and Integration

$f(x)$	$f'(x)$
$\cot x$	$-\operatorname{cosec}^2 x$
$\sec x$	$\sec x \tan x$
$\operatorname{cosec} x$	$-\operatorname{cosec} x \cot x$

$f(x)$	$f'(x)$
$\coth x$	$-\operatorname{cosech}^2 x$
$\operatorname{sech} x$	$-\operatorname{sech} x \tanh x$
$\operatorname{cosech} x$	$-\operatorname{cosech} x \coth x$

$$\int \frac{f'(x)}{f(x)}\,dx = \ln|f(x)| + c$$

1. Find $\int \sinh t \, e^{\cosh t} \, dt$.

[4 marks]

2. Solve the equation $2\tan^2\theta = 5\sec\theta + 1$ for $0° \le \theta \le 360°$.

[5 marks]

3. The curve $y = x^3 + px^2 + qx + r$ passes through the point $(0,5)$ and has stationary points at $x = 4$ and $x = -3$. Find the values of p, q and r.

[6 marks]

4. A circle passes through the point $(5,2)$ and touches the line $y + x = 9$ at the point $(3,6)$. Find the coordinates of the center and the radius of the circle. Hence, state the standard equation of the circle.

[7 marks]

5. If $y = \ln(\cos x + 1)$, show that $\sin x \dfrac{d^2y}{dx^2} - \dfrac{dy}{dx} = 0$.

[7 marks]

6. Given the parametric equations $x = 2\cos\theta + 1$ and $y = 3\sin\theta + 2$.

   (a) Show that the above parametric equations represent an ellipse.

   (Hint : $\sin^2 x + \cos^2 x = 1$)

   [3 marks]

   (b) Find the center, foci and vertices of the ellipse.

   [5 marks]

   (c) Sketch the graph of the ellipse.

   [2 marks]

7. Express $12\cos\theta - 5\sin\theta$ in the form of $R\cos(\theta + \alpha)$ where $R > 0$ and $0° < \alpha < 90°$.

[5 marks]

The function f is defined by $f(\theta) = 12\cos\theta - 5\sin\theta + 10$.

(a) Find the maximum and minimum values of $f(\theta)$.

[3 marks]

(b) Find the value of θ for $0° < \theta < 360°$ such that $f(\theta)$ is maximum.

[3 marks]

END OF BOOKLET