



KEMENTERIAN
PENDIDIKAN
MALAYSIA

QS 015/1

**Matriculation Programme
Examination**

Semester I

Session 2013/2014

1. Given matrices $A = \begin{bmatrix} 5 & 3 & 0 \\ 3 & -1 & 0 \\ 0 & 0 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} c & 3 & 0 \\ 3 & d & 0 \\ 0 & 0 & e \end{bmatrix}$. Find the values of c, d and e such that $AB = 14I$, where I is the identity matrix. Hence, determine A^{-1} .
2. Consider the function $f(x) = 1 + \ln(x)$, $x \geq 1$. Determine $f^{-1}(x)$ and state its range. Hence, evaluate $f^{-1}(3)$.
3. Find the value of x which satisfies the equation $\log_9 x = (\log_3 x)^2$, $x > 1$.
4. Solve the equation $2^{2x-2} - 2^{x+1} = 2^x - 2^3$.
5. Given $g(x) = \frac{kx+8}{4x-5}$, $x \neq \frac{5}{4}$ where k is a constant.
- Find the value of k if $(g \circ g)(x) = x$.
 - Find the value of k so that $g(x)$ is not a one-to-one function.
6. Given $f(x) = e^{3x} + 4$, $x \in \mathbb{R}$.
- Find $f^{-1}(x)$.
 - On the same axes, sketch the graphs of $f(x)$ and $f^{-1}(x)$. State the domain of $f(x)$ and $f^{-1}(x)$.
7. a. Find the values of p and q if $\frac{p}{4-2i} + \frac{q}{4+2i} = 1 + \frac{5}{2}i$.
- b. Given $\log_{10} 2 = m$ and $\log_{10} 7 = n$. Express x in terms of m and n if $(14^{3x+1})(8^{2x+3}) = 7$.

8. An osteoporosis patient was advised by a doctor to take enough magnesium, vitamin D and calcium to improve bone density. In a week, the patient has to take 8 units magnesium, 11 units vitamin D and 17 units calcium. The following are three types of capsule that contains the three essential nutrients for the bone:

Capsule of type P: 2 units magnesium, 1 unit vitamin D and 1 unit calcium.

Capsule of type Q: 1 unit magnesium, 2 units vitamin D and 3 units calcium.

Capsule of type R: 4 units magnesium, 6 units vitamin D and 10 units calcium.

Let x , y and z represent the number of capsule of types P, Q and R respectively that the patient has to take in a week.

- (a) Obtain a system of linear equation to represent the given information and write the system in the form of matrix equation $AX = B$, where $X = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$.

- (b) Find the inverse of matrix A from part (a) by using the adjoint method. Hence, find the values of x , y and z .

- (c) The cost for each capsule of type P, Q and R are RM10, RM15 and RM17 respectively. How much will the expenses be for 4 weeks if the patient follows the doctor's advice?

9. a. In an arithmetic progression, the sum of the first four terms is 46 and the seventh term exceeds twice of the second term by 5. Obtain the first term and the common difference for the progression. Hence, calculate the sum of the first ten even terms of the progression.

- b. A ball is dropped from a height of 2m. Each time the ball hits the floor, it bounces vertically to a height that is $\frac{3}{4}$ of its previous height.

- i. Find the height of the ball at the tenth bounce.
- ii. Find the total distance that the ball will travel before the eleventh bounce.

- 10 a. Find the solution set of $|2 - 3x| < |x + 3|$.

- b. If $x + 1 < 0$, show that

- i. $2x - 1 < 0$
- ii. $\frac{2x-1}{x+1} > 2$

1. Given matrices $A = \begin{bmatrix} 5 & 3 & 0 \\ 3 & -1 & 0 \\ 0 & 0 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} c & 3 & 0 \\ 3 & d & 0 \\ 0 & 0 & e \end{bmatrix}$. Find the values of c, d and e such that $AB = 14I$, where I is the identity matrix. Hence, determine A^{-1} .

SOLUTION

$$AB = 14I$$

$$\begin{bmatrix} 5 & 3 & 0 \\ 3 & -1 & 0 \\ 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} c & 3 & 0 \\ 3 & d & 0 \\ 0 & 0 & e \end{bmatrix} = \begin{bmatrix} 14 & 0 & 0 \\ 0 & 14 & 0 \\ 0 & 0 & 14 \end{bmatrix}$$

$$\begin{bmatrix} 5c + 9 & 15 + 3d & 0 \\ 3c - 3 & 9 - d & 0 \\ 0 & 0 & 2e \end{bmatrix} = \begin{bmatrix} 14 & 0 & 0 \\ 0 & 14 & 0 \\ 0 & 0 & 14 \end{bmatrix}$$

$$5c + 9 = 14 \quad \rightarrow \quad c = 1$$

$$15 + 3d = 0 \quad \rightarrow \quad d = -5$$

$$2e = 14 \quad \rightarrow \quad e = 7$$

$$\therefore c = 1, d = -5, e = 7$$

$$AB = 14I$$

$$A^{-1} = \frac{1}{14} B$$

$$A^{-1} = \frac{1}{14} \begin{bmatrix} 1 & 3 & 0 \\ 3 & -5 & 0 \\ 0 & 0 & 7 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} \frac{1}{14} & \frac{3}{14} & 0 \\ \frac{3}{14} & \frac{-5}{14} & 0 \\ 0 & 0 & \frac{7}{14} \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} \frac{1}{14} & \frac{3}{14} & 0 \\ \frac{3}{14} & \frac{-5}{14} & 0 \\ 0 & 0 & \frac{1}{2} \end{bmatrix}$$

2. Consider the function $f(x) = 1 + \ln(x)$, $x \geq 1$. Determine $f^{-1}(x)$ and state its range. Hence, evaluate $f^{-1}(3)$.

SOLUTION

<i>Method I</i>	<i>Method II</i>
$y = \ln x + 1, \quad x \geq 1$ $\ln x = y - 1$ $x = e^{y-1}$ $f^{-1}(x) = e^{x-1}$ $R_{f^{-1}} = \{y: y \geq 1\}$ $f^{-1}(3) = e^{3-1} = e^2$	$f(x) = 1 + \ln(x)$ $f(f^{-1}(x)) = x$ $1 + \ln(f^{-1}(x)) = x$ $\ln(f^{-1}(x)) = x - 1$ $f^{-1}(x) = e^{x-1}$ $R_{f^{-1}} = \{y: y \geq 1\}$ $f^{-1}(3) = e^{3-1} = e^2$

3. Find the value of x which satisfies the equation

$$\log_9 x = (\log_3 x)^2, \quad x > 1.$$

SOLUTION

$$\log_9 x = (\log_3 x)^2$$

$$\frac{\log_3 x}{\log_3 9} = (\log_3 x)^2$$

$$\frac{\log_3 x}{\log_3 3^2} = (\log_3 x)^2$$

$$\frac{\log_3 x}{2\log_3 3} = (\log_3 x)^2$$

$$\log_3 x = 2(\log_3 x)^2$$

$$\text{Let } u = \log_3 x$$

$$u = 2u^2$$

$$2u^2 - u = 0$$

$$u(2u - 1) = 0$$

$$u = 0 \quad \text{or} \quad (2u - 1) = 0$$

$$u = 0 \quad \text{or} \quad u = \frac{1}{2}$$

$$\log_3 x = 0 \quad \text{or} \quad \log_3 x = \frac{1}{2}$$

$$x = 3^0 \quad \text{or} \quad x = 3^{\frac{1}{2}}$$

$$x = 1 \quad \text{or} \quad x = \sqrt{3}$$

Since $x > 1$, the solution is $x = \sqrt{3}$

4. Solve the equation $2^{2x-2} - 2^{x+1} = 2^x - 2^3$.

SOLUTION

$$2^{2x-2} - 2^{x+1} = 2^x - 2^3$$

$$2^{2x}2^{-2} - 2^x2^1 = 2^x - 2^3$$

$$\frac{(2^x)^2}{4} - 2(2^x) = 2^x - 8$$

$$\text{Let } u = 2^x$$

$$\frac{(u)^2}{4} - 2(u) = u - 8$$

$$u^2 - 8u = 4u - 32$$

$$u^2 - 8u - 4u + 32 = 0$$

$$u^2 - 12u + 32 = 0$$

$$(u - 8)(u - 4) = 0$$

$$u - 8 = 0 \quad \text{or} \quad u - 4 = 0$$

$$u = 8 \quad \text{or} \quad u = 4$$

$$2^x = 8 \quad \text{or} \quad 2^x = 4$$

$$x = 3 \quad \text{or} \quad x = 2$$

5. Given $g(x) = \frac{kx+8}{4x-5}$, $x \neq \frac{5}{4}$ where k is a constant.
- Find the value of k if $(g \circ g)(x) = x$.
 - Find the value of k so that $g(x)$ is not a one-to-one function.

SOLUTION**5a.**

$$g(x) = \frac{kx+8}{4x-5}$$

$$(g \circ g)(x) = x$$

$$g[g(x)] = x$$

$$\frac{k\left(\frac{kx+8}{4x-5}\right) + 8}{4\left(\frac{kx+8}{4x-5}\right) - 5} = x$$

$$\frac{\frac{k(kx+8)}{4x-5} + \frac{8(4x-5)}{(4x-5)}}{\frac{4(kx+8)}{4x-5} - \frac{5(4x-5)}{(4x-5)}} = x$$

$$\frac{\frac{k(kx+8) + 8(4x-5)}{4x-5}}{\frac{4(kx+8) - 5(4x-5)}{4x-5}} = x$$

$$\frac{k(kx+8) + 8(4x-5)}{4(kx+8) - 5(4x-5)} = x$$

$$(k^2x + 8k) + (32x - 40) = x[(4kx + 32) - (20x - 25)]$$

$$k^2x + 8k + 32x - 40 = x[4kx + 32 - 20x + 25]$$

$$(k^2 + 32)x + (8k - 40) = 4kx^2 + 32x - 20x^2 + 25x$$

$$(k^2 + 32)x + (8k - 40) = (4k - 20)x^2 + 57x$$

Compare the coefficient of x^2

$$4k - 20 = 0$$

$$k = 5$$

5b.

$$g(x) = \frac{kx + 8}{4x - 5}$$

Let $x_2 \neq x_1$, but $g(x_2) = g(x_1)$

$$\frac{kx_2 + 8}{4x_2 - 5} = \frac{kx_1 + 8}{4x_1 - 5}$$

$$(kx_2 + 8)(4x_1 - 5) = (kx_1 + 8)(4x_2 - 5)$$

$$4kx_1x_2 - 5kx_2 + 32x_1 - 40 = 4kx_1x_2 - 5kx_1 + 32x_2 - 40$$

$$32x_1 - 5kx_2 - 32x_2 + 5kx_1 = 0$$

$$32x_1 - 32x_2 + 5kx_1 - 5kx_2 = 0$$

$$32(x_1 - x_2) + 5k(x_1 - x_2) = 0$$

$$(x_1 - x_2)(32 + 5k) = 0$$

$$x_1 - x_2 \neq 0 \rightarrow 32 + 5k = 0$$

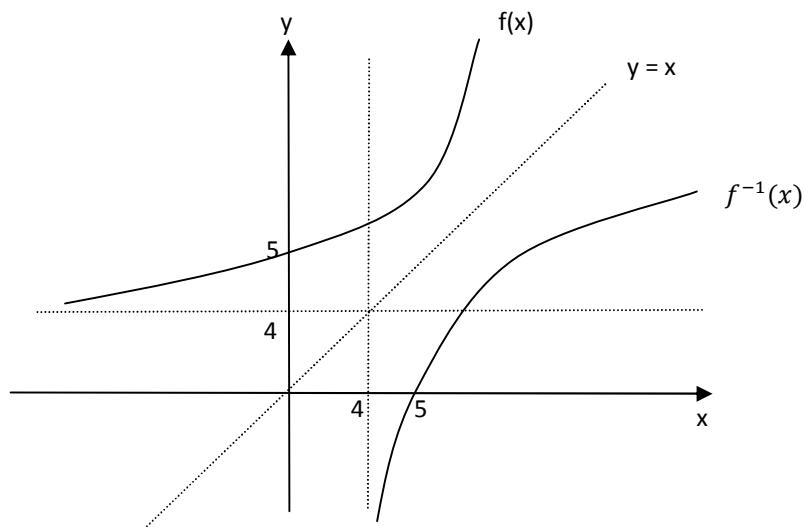
$$k = \frac{-32}{5}$$

6. Given $f(x) = e^{3x} + 4$, $x \in \mathfrak{R}$.
- Find $f^{-1}(x)$.
 - On the same axes, sketch the graphs of $f(x)$ and $f^{-1}(x)$. State the domain of $f(x)$ and $f^{-1}(x)$.

SOLUTION**6a**

$$f(x) = e^{3x} + 4$$

Method I	Method II
<p>Let $y = e^{3x} + 4$</p> $e^{3x} = y - 4$ $\ln(e^{3x}) = \ln(y - 4)$ $3x \ln e = \ln(y - 4)$ $3x = \ln(y - 4)$ $x = \frac{1}{3} \ln(y - 4)$ $\therefore f^{-1}(x) = \frac{1}{3} \ln(x - 4)$	$f[f^{-1}(x)] = x$ $e^{3f^{-1}(x)} + 4 = x$ $e^{3f^{-1}(x)} = x - 4$ $\ln e^{3f^{-1}(x)} = \ln(x - 4)$ $3f^{-1}(x) \ln e = \ln(x - 4)$ $3f^{-1}(x) = \ln(x - 4)$ $f^{-1}(x) = \frac{1}{3} \ln(x - 4)$

6bDomain $f(x): \{-\infty, \infty\}$ Domain $f^{-1}(x): \{4, \infty\}$

7. a. Find the values of p and q if $\frac{p}{4-2i} + \frac{q}{4+2i} = 1 + \frac{5}{2}i$.
 b. Given $\log_{10}2 = m$ and $\log_{10}7 = n$. Express x in terms of m and n if $(14^{3x+1})(8^{2x+3}) = 7$.

SOLUTION

7a

$$\frac{p}{4-2i} + \frac{q}{4+2i} = 1 + \frac{5}{2} i$$

$$\frac{p(4+2i) + q(4-2i)}{(4-2i)(4+2i)} = \frac{2+5i}{2}$$

$$\frac{4p + 2pi + 4q - 2qi}{16 + 8i - 8i - 4i^2} = \frac{2 + 5i}{2}$$

$$\frac{4p + 4q + 2pi - 2qi}{16 - 4(-1)} = \frac{2 + 5i}{2}$$

$$\frac{(4p + 4q) + (2p - 2q)i}{20} = \frac{2 + 5i}{2}$$

$$2[(4p + 4q) + (2p - 2q)i] = 20[2 + 5i]$$

$$[(4p + 4q) + (2p - 2q)i] = \frac{20}{2} [2 + 5i]$$

$$[(4p + 4q) + (2p - 2q)i] = 10[2 + 5i]$$

$$[(4p + 4q) + (2p - 2q)i] = [20 + 50i]$$

$$(4p + 4q) = 20$$

$$2p - 2q = 50$$

$$p - q = 25 \dots \dots \dots \quad (2)$$

(1) + (2)

$$2p = 30$$

p = 15

$$g = -10$$

7b

$$\log_{10} 2 = m, \quad \log_{10} 7 = n$$

$$(14^{3x+1})(8^{2x+3}) = 7$$

$$[(2x7)^{3x+1}][(2^3)^{2x+3}] = 7$$

$$[2^{3x+1}x7^{3x+1}][(2)^{6x+9}] = 7$$

$$[2^{3x+1}x(2)^{6x+9}][7^{3x+1}] = 7$$

$$[2^{3x+1+6x+9}][7^{3x+1}] = 7$$

$$[2^{9x+10}][7^{3x+1}] = 7$$

$$\log\{[2^{9x+10}][7^{3x+1}]\} = \log 7$$

$$\log 2^{9x+10} + \log 7^{3x+1} = \log 7$$

$$(9x+10)\log 2 + (3x+1)\log 7 = \log 7$$

$$(9x+10)m + (3x+1)n = n$$

$$9xm + 10m + 3xn + n = n$$

$$9xm + 3xn = n - n - 10m$$

$$x(9m + 3n) = -10m$$

$$x = \frac{-10m}{9m + 3n}$$

8. An osteoporosis patient was advised by a doctor to take enough magnesium, vitamin D and calcium to improve bone density. In a week, the patient has to take 8 units magnesium, 11 units vitamin D and 17 units calcium. The following are three types of capsule that contains the three essential nutrients for the bone:

Capsule of type P: 2 units magnesium, 1 unit vitamin D and 1 unit calcium.

Capsule of type Q: 1 unit magnesium, 2 units vitamin D and 3 units calcium.

Capsule of type R: 4 units magnesium, 6 units vitamin D and 10 units calcium.

Let x , y and z represent the number of capsule of types P, Q and R respectively that the patient has to take in a week.

- (a) Obtain a system of linear equation to represent the given information and write the system in the form of matrix equation $AX = B$, where $X = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$.
- (b) Find the inverse of matrix A from part (a) by using the adjoint method. Hence, find the values of x , y and z .
- (c) The cost for each capsule of type P, Q and R are RM10, RM15 and RM17 respectively. How much will the expenses be for 4 weeks if the patient follows the doctor's advice?

SOLUTION

8a

$$2x + y + 4z = 8$$

$$x + 2y + 6z = 11$$

$$x + 3y + 10z = 17$$

$$\begin{bmatrix} 2 & 1 & 4 \\ 1 & 2 & 6 \\ 1 & 3 & 10 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 8 \\ 11 \\ 17 \end{bmatrix}$$

8b

$$A = \begin{bmatrix} 2 & 1 & 4 \\ 1 & 2 & 6 \\ 1 & 3 & 10 \end{bmatrix}$$

$$|A| = (2) \begin{vmatrix} 2 & 6 \\ 3 & 10 \end{vmatrix} - (1) \begin{vmatrix} 1 & 6 \\ 1 & 10 \end{vmatrix} + (4) \begin{vmatrix} 1 & 2 \\ 1 & 3 \end{vmatrix}$$

$$|A| = (2)[20 - 18] - (1)[10 - 6] + (4)[3 - 2]$$

$$|A| = (2)[2] - (1)[4] + (4)[1]$$

$$|A| = 4$$

$$\text{Cofactor of } A = \begin{bmatrix} +|2 & 6| & -|1 & 6| & +|1 & 2| \\ -|1 & 4| & +|2 & 4| & -|2 & 1| \\ +|1 & 4| & -|2 & 4| & +|2 & 1| \end{bmatrix}$$

$$\text{Cofactor of } A = \begin{bmatrix} +(20 - 18) & -(10 - 6) & +(3 - 2) \\ -(10 - 12) & +(20 - 4) & -(6 - 1) \\ +(6 - 8) & -(12 - 4) & +(4 - 1) \end{bmatrix}$$

$$\text{Cofactor of } A = \begin{bmatrix} 2 & -4 & 1 \\ 2 & 16 & -5 \\ -2 & -8 & 3 \end{bmatrix}$$

$$\text{Adjoin of } A = C^T$$

$$\text{Adjoin of } A = \begin{bmatrix} 2 & 2 & -2 \\ -4 & 16 & -8 \\ 1 & -5 & 3 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} \text{Adj } A$$

$$A^{-1} = \frac{1}{4} \begin{bmatrix} 2 & 2 & -2 \\ -4 & 16 & -8 \\ 1 & -5 & 3 \end{bmatrix}$$

$$AX = B$$

$$A^{-1}AX = A^{-1}B$$

$$X = A^{-1}B$$

$$X = \frac{1}{4} \begin{bmatrix} 2 & 2 & -2 \\ -4 & 16 & -8 \\ 1 & -5 & 3 \end{bmatrix} \begin{bmatrix} 8 \\ 11 \\ 17 \end{bmatrix}$$

$$X = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$$

$$\therefore x = 1, \quad y = 2, \quad z = 1$$

8c

$$P = 10, \quad Q = 15, \quad R = 17$$

$$\text{Expenses for 4 weeks} = 4[10(1) + 15(2) + 17(1)]$$

$$\text{Expenses for 4 weeks} = 4[57]$$

$$\text{Expenses for 4 weeks} = RM228$$

9. a. In an arithmetic progression, the sum of the first four terms is 46 and the seventh term exceeds twice of the second term by 5. Obtain the first term and the common difference for the progression. Hence, calculate the sum of the first ten even terms of the progression.

b. A ball is dropped from a height of 2m. Each time the ball hits the floor, it bounces vertically to a height that is $\frac{3}{4}$ of its previous height.

 - i. Find the height of the ball at the tenth bounce.
 - ii. Find the total distance that the ball will travel before the eleventh bounce.

SOLUTION

9a

Arithmetic progression $S_n = \frac{n}{2}[2a + (n - 1)d]$, $T_n = a + (n - 1)d$

$$S_4 = 46$$

$$\frac{4}{2}[2a + (4 - 1)d] = 46$$

$$2[2a + 3d] = 46$$

$$4a + 6d = 46$$

$$T_7 = 2T_2 + 5$$

$$a + (7 - 1)d = 2[a + (2 - 1)d] + 5$$

$$a + 6d = 2[a + d] + 5$$

$$a + 6d = 2a + 2d + 5$$

$$a - 4d = -5$$

$$2a - 8d = -10 \quad \dots \dots \dots \quad (2)$$

$$(1) - (2)$$

$$11d = 33$$

d = 3

a = 7

9b(i)

$$1^{\text{st}} \text{ bounce} \Rightarrow T_1 = 2 \left(\frac{3}{4} \right)$$

$$2^{\text{nd}} \text{ bounce} \Rightarrow T_2 = 2 \left(\frac{3}{4} \right) \left(\frac{3}{4} \right) = 2 \left(\frac{3}{4} \right)^2$$

$$3^{\text{rd}} \text{ bounce} \Rightarrow T_3 = 2 \left(\frac{3}{4} \right)^3$$

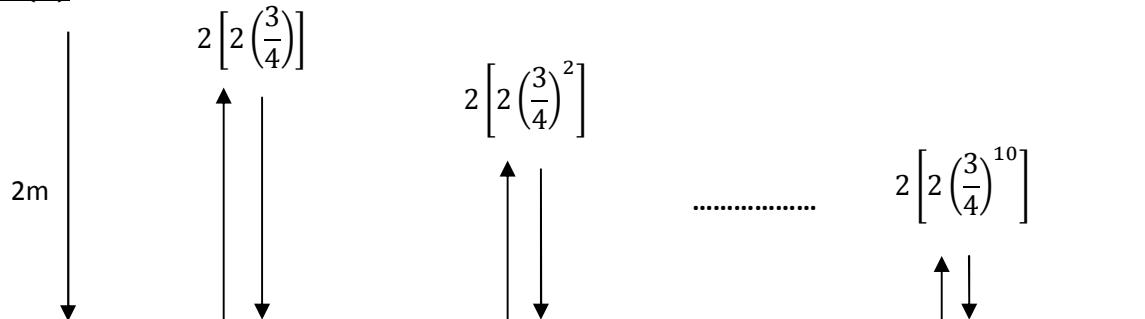
$$a = 2 \left(\frac{3}{4} \right), \quad r = \left(\frac{3}{4} \right)$$

$$T_r = ar^{n-1}$$

$$T_{10} = \left[2 \left(\frac{3}{4} \right) \right] \left[\left(\frac{3}{4} \right)^{10-1} \right]$$

$$T_{10} = \left[2 \left(\frac{3}{4} \right) \right] \left[\left(\frac{3}{4} \right)^9 \right]$$

$$T_{10} = 2 \left(\frac{3}{4} \right)^{10}$$

9b(ii)

$$\text{Total distance} = 2 + 2 \left[2 \left(\frac{3}{4} \right) \right] + 2 \left[2 \left(\frac{3}{4} \right)^2 \right] + \dots + 2 \left[2 \left(\frac{3}{4} \right)^{10} \right]$$

$$= 2 \left[1 + 2 \left(\frac{3}{4} \right) + 2 \left(\frac{3}{4} \right)^2 + \dots + 2 \left(\frac{3}{4} \right)^{10} \right]$$

$$= 2 \left\{ 1 + 2 \underbrace{\left[\left(\frac{3}{4} \right) + \left(\frac{3}{4} \right)^2 + \dots + \left(\frac{3}{4} \right)^{10} \right]} \right\}$$

$$= 2 \left\{ 1 + 2 \left[\frac{\left(\frac{3}{4} \right) \left[1 - \left(\frac{3}{4} \right)^{10} \right]}{1 - \left(\frac{3}{4} \right)} \right] \right\}$$

$$= 13.324$$

$$\boxed{a = \left(\frac{3}{4} \right), \quad r = \left(\frac{3}{4} \right)}$$

$$S_n = \frac{a(1 - r^n)}{1 - r}$$

10 a. Find the solution set of $|2 - 3x| < |x + 3|$.

b. If $x + 1 < 0$, show that

- i. $2x - 1 < 0$
- ii. $\frac{2x-1}{x+1} > 2$

SOLUTION

10a

$$|2 - 3x| < |x + 3|$$

$$(2 - 3x)^2 < (x + 3)^2$$

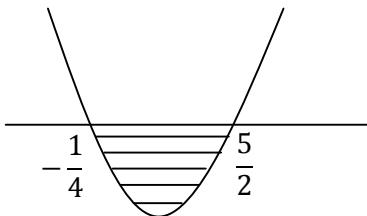
$$4 - 12x + 9x^2 < x^2 + 6x + 9$$

$$9x^2 - x^2 - 12x - 6x + 4 - 9 < 0$$

$$8x^2 - 18x - 5 < 0$$

$$(2x - 5)(4x + 1) < 0$$

$$x = \frac{5}{2}, -\frac{1}{4}$$



$$\therefore \text{Solution set is } \left\{ x : -\frac{1}{4} < x < \frac{5}{2} \right\}$$

10b(i)

$$2x - 1 = 2x + 2 - 3 = 2(x + 1) - 3$$

$$\because x + 1 < 0 \rightarrow 2(x + 1) - 3 < 0$$

$$\therefore 2x - 1 < 0$$

10b(ii)

$$\frac{2x - 1}{x + 1} > 2$$

$$\frac{2x - 1}{x + 1} - 2 > 0$$

$$\frac{2x - 1}{x + 1} - 2 = \frac{(2x - 1) - 2(x + 1)}{x + 1}$$

$$= \frac{2x - 1 - 2x - 2}{x + 1}$$

$$= \frac{-3}{x + 1}$$

$$\because x + 1 < 0$$

$$\frac{-3}{x + 1} > 0$$

$$\frac{2x - 1}{x + 1} - 2 > 0$$

$$\therefore \frac{2x - 1}{x + 1} > 2$$