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QS 015

Mid-Semester Examination

Semester I

Session 2014/2015

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1. If $\begin{vmatrix} -2 & 1 & x+2 \\ 3 & x-4 & 5 \\ 0 & 1 & 3 \end{vmatrix}$ = 25, find the value of x.

2. Given a complex number $z = \frac{2i}{\sqrt{3}+i}$.

   (a) State z in the form of a + bi, where a and b are real numbers.

   (b) Find the modulus and argument of z.

3. (a) Simplify $\left(3\sqrt{2}+\ 1\right)^2$ in the form $a + b\sqrt{c}$.

   (b) If $2\log x + 3\log y = 0$, find y in terms of x.

4. Solve $\frac{3x}{x-4} \geq \frac{2x}{7}$.

5. Expand $(1-x)^{\frac{1}{2}}$ in ascending powers of x up to and including the term in x^3. State the range of x for which the expansion is valid.

   Hence,

   (a) Show that $(9-2x)^{\frac{1}{2}} \approx 3 - \frac{1}{3}x - \frac{1}{54}x^2 - \frac{1}{486}x^3 + \ \ldots$

   (b) By substituting $x = \frac{1}{2}$ in the expansion in part (a), find the value of $\sqrt{8}$ correct to four decimal places.

6. Given a matrix $A = \begin{bmatrix} 1 & 1 & x \\ 3 & 2 & 5 \\ 2 & 1 & 1 \end{bmatrix}$.

   (a) If $M_{31} = -5$, find the value of x.

   (b) Hence, find

      (i) $|A|$.

      (ii) the values of a, b and c if the cofactor matrix of A is

      $$\begin{bmatrix} -3 & a & -1 \\ b & -9 & 1 \\ -5 & 10 & c \end{bmatrix}.$$

      (iii) A^{-1}.

1. If $\begin{vmatrix} -2 & 1 & x+2 \\ 3 & x-4 & 5 \\ 0 & 1 & 3 \end{vmatrix}$ = 25, find the value of x.

Solution

$$\begin{vmatrix} -2 & 1 & x+2 \\ 3 & x-4 & 5 \\ 0 & 1 & 3 \end{vmatrix} = +(0)\begin{vmatrix} 1 & x+2 \\ x-4 & 5 \end{vmatrix} - (1)\begin{vmatrix} -2 & x+2 \\ 3 & 5 \end{vmatrix} + (3)\begin{vmatrix} -2 & 1 \\ 3 & x-4 \end{vmatrix}$$

$$= 0 - [(-2)(5) - (3)(x+2)] + 3[(-2)(x-4) - (3)(1)]$$

$$= 0 - [(-10) - (3x+6)] + 3[(-2x+8) - (3)]$$

$$= 0 - [-10 - 3x - 6] + 3[-2x + 8 - 3]$$

$$= 0 - [-16 - 3x] + 3[-2x + 5]$$

$$= 16 + 3x - 6x + 15$$

$$= 31 - 3x$$

Since $\begin{vmatrix} -2 & 1 & x+2 \\ 3 & x-4 & 5 \\ 0 & 1 & 3 \end{vmatrix}$ = 25

$$31 - 3x = 25$$

$$3x = 31 - 25$$

$$3x = 6$$

$$x = 2$$

2. Given a complex number $z = \frac{2i}{\sqrt{3}+i}$.

(a) State z in the form of a + bi, where a and b are real numbers.

(b) Find the modulus and argument of z.

Solution

(a) $z = \frac{2i}{\sqrt{3}+i}$

$$= \frac{2i}{\sqrt{3}+i} \cdot \frac{\sqrt{3}-i}{\sqrt{3}-i}$$

$$= \frac{2\sqrt{3}i - 2i^2}{3+1}$$

$$= \frac{2\sqrt{3}i + 2}{4}$$

$$= \frac{2[\sqrt{3}i + 1]}{4}$$

$$= \frac{[\sqrt{3}i + 1]}{2}$$

$$= \frac{\sqrt{3}i}{2} + \frac{1}{2}$$

$$= \frac{1}{2} + \frac{\sqrt{3}i}{2}$$

(b) $|z| = \sqrt{\left(\frac{\sqrt{3}}{2}\right)^2 + \left(\frac{1}{2}\right)^2}$

$$= \sqrt{\frac{3}{4} + \frac{1}{4}}$$

$$= \sqrt{\frac{4}{4}}$$

$$= 1$$

$$\alpha = tan^{-1}\left[\frac{b}{a}\right]$$

$$= tan^{-1}\left[\frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}}\right]$$

$$= tan^{-1}\left[\frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}}\right]$$

$$= tan^{-1}\left[\frac{\sqrt{3}}{2} \cdot \frac{2}{1}\right]$$

$$= tan^{-1}\left[\sqrt{3}\right]$$

$$= \frac{\pi}{3} \quad or \quad 1.047$$

$$\arg(z), \theta = \pi - \frac{\pi}{3}$$

$$= \frac{2\pi}{3} \quad or \quad 2.09$$

3. (a) Simplify $\left(3\sqrt{2}+1\right)^2$ in the form $a+b\sqrt{c}$.

(b) If $2\log x+3\log y=0$, find y in terms of x.

Solution

(a) $\left(3\sqrt{2}+1\right)^2 = \left(3\sqrt{2}+1\right)\left(3\sqrt{2}+1\right)$

$= \left(3\sqrt{2}\right)^2 + 2\left(3\sqrt{2}\right) + 1$

$= 18 + 6\sqrt{2} + 1$

$= 19 + 6\sqrt{2}$

(b) $2\log x+3\log y=0$

$\log x^2+\log y^3=0$

$\log x^2y^3=0$

$x^2y^3=1$

$y^3=\dfrac{1}{x^2}$

$y=x^{-\frac{2}{3}}$

Alternative

$2\log x+3\log y=0$

$2\log x=-3\log y$

$\log x^2=\log y^{-3}$

$x^2=y^{-3}$

$y^{-3}=x^2$

$(y^{-3})^{-\frac{1}{3}}=(x^2)^{-\frac{1}{3}}$

$y=x^{-\frac{2}{3}}$

4. Solve $\frac{3x}{x-4} \geq \frac{2x}{7}$.

Solution

$$\frac{3x}{x-4} \geq \frac{2x}{7}$$

$$\frac{3x}{x-4} - \frac{2x}{7} \geq 0$$

$$\frac{7(3x) - 2x(x-4)}{7(x-4)} \geq 0$$

$$\frac{21x - 2x^2 + 8x}{7(x-4)} \geq 0$$

$$\frac{29x - 2x^2}{7(x-4)} \geq 0$$

$$\frac{x(29-2x)}{7(x-4)} \geq 0$$

Crictical Values:

$$x = 0; \quad x = \frac{29}{2}; \quad x = 4$$

	$(-\infty, \mathbf{0})$	$(\mathbf{0}, \mathbf{4})$	$\left(\mathbf{4}, \frac{\mathbf{29}}{\mathbf{2}}\right)$	$\left(\frac{\mathbf{29}}{\mathbf{2}}, \infty\right)$
x	-	+	+	+
$(29-2x)$	+	+	+	-
$(x-4)$	-	-	+	+
$\frac{x(29-2x)}{7(x-4)}$	⊕	-	⊕	-

$$\therefore \ (-\infty, \mathbf{0}] \cup \ \left(\mathbf{4}, \frac{\mathbf{29}}{\mathbf{2}}\right]$$

5. Expand $(1-x)^{\frac{1}{2}}$ in ascending powers of x up to and including the term in x^3. State the range of x for which the expansion is valid.

Hence,

(a) Show that $(9-2x)^{\frac{1}{2}} \approx 3 - \frac{1}{3}x - \frac{1}{54}x^2 - \frac{1}{486}x^3 + \ldots$

(b) By substituting $x = \frac{1}{2}$ in the expansion in part (a), find the value of $\sqrt{8}$ correct to four decimal places.

Solution

$$(1-x)^{\frac{1}{2}} = 1 + \left(\frac{1}{2}\right)(-x) + \frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)}{2!}(-x)^2 + \frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{3!}(-x)^3 + \ldots$$

$$= 1 - \frac{1}{2}x - \frac{1}{8}x^2 - \frac{1}{16}x^3 + \ldots$$

the range of x for which the expansion is valid

$$|x| < 1$$

$$-1 < x < 1$$

(a) $(9-2x)^{\frac{1}{2}} = \left[9\left(1-\frac{2}{9}x\right)\right]^{\frac{1}{2}}$

$$= 3\left(1-\frac{2}{9}x\right)^{\frac{1}{2}}$$

$$= 3\left[1 - \frac{1}{2}\left(\frac{2}{9}x\right) - \frac{1}{8}\left(\frac{2}{9}x\right)^2 - \frac{1}{16}\left(\frac{2}{9}x\right)^3 + \ldots\right]$$

$$= 3\left[1 - \frac{1}{9}x - \frac{1}{162}x^2 - \frac{1}{1458}x^3 + \ldots\right]$$

$$= 3 - \frac{1}{3}x - \frac{1}{54}x^2 - \frac{1}{486}x^3 + \ldots$$

(b) $(9-2x)^{\frac{1}{2}} = 3 - \frac{1}{3}x - \frac{1}{54}x^2 - \frac{1}{486}x^3 + \ldots$

$$\left[9 - 2\left(\frac{1}{2}\right)\right]^{\frac{1}{2}} == 3 - \frac{1}{3}\left(\frac{1}{2}\right) - \frac{1}{54}\left(\frac{1}{2}\right)^2 - \frac{1}{486}\left(\frac{1}{2}\right)^3 + \ldots$$

$$\sqrt{8} = 2.8284$$

6. Given a matrix $A = \begin{bmatrix} 1 & 1 & x \\ 3 & 2 & 5 \\ 2 & 1 & 1 \end{bmatrix}$.

(a) If $M_{31} = -5$, find the value of x.

(b) Hence, find

(i) $|A|$.

(ii) the values of a, b and c if the cofactor matrix of A is

$$\begin{bmatrix} -3 & a & -1 \\ b & -9 & 1 \\ -5 & 10 & c \end{bmatrix}.$$

(iii) A^{-1}.

Solution

$$A = \begin{bmatrix} 1 & 1 & x \\ 3 & 2 & 5 \\ 2 & 1 & 1 \end{bmatrix}$$

(a) $M_{31} = -5$

$$\begin{vmatrix} 1 & x \\ 2 & 5 \end{vmatrix} = -5$$

$$5 - 2x = -5$$

$$2x = 10$$

$$x = 5$$

(b) (i) $A = \begin{bmatrix} 1 & 1 & 5 \\ 3 & 2 & 5 \\ 2 & 1 & 1 \end{bmatrix}$

$$|A| = +(1)\begin{vmatrix} 2 & 5 \\ 1 & 1 \end{vmatrix} - (1)\begin{vmatrix} 3 & 5 \\ 2 & 1 \end{vmatrix} + (5)\begin{vmatrix} 3 & 2 \\ 2 & 1 \end{vmatrix}$$

$$|A| = (1)\,[(2)(1) - (5)(1)] - (1)[(3)(1) - (5)(2)] + (5)[(3)(1) - (2)(2)]$$

$$|A| = (1)\,[2 - 5] - (1)[3 - 10] + (5)[3 - 4]$$

$$|A| = (1)\,[-3] - (1)[-7] + (5)[-1]$$

$$|A| = -3 + 7 - 5$$

$$|A| = -1$$

(b) (ii) $C_{12} = -\begin{vmatrix} 3 & 5 \\ 2 & 1 \end{vmatrix}$

$= -[(3)(1) - (5)(2)]$

$= -[3 - 10]$

$= 7$

Given that $C_{12} = a$

$\therefore a = 7$

$C_{21} = -\begin{vmatrix} 1 & 5 \\ 1 & 1 \end{vmatrix}$

$= -[(1)(1) - (5)(1)]$

$= -[1 - 5]$

$= 4$

Given that $C_{21} = b$

$\therefore b = 7$

$C_{33} = \begin{vmatrix} 1 & 1 \\ 3 & 2 \end{vmatrix}$

$= -[(1)(2) - (1)(3)]$

$= -[2 - 3]$

$= -1$

Given that $C_{33} = c$

$\therefore c = -1$

(b) (iii) $A = \begin{bmatrix} 1 & 1 & 5 \\ 3 & 2 & 5 \\ 2 & 1 & 1 \end{bmatrix}$

$$C = \begin{bmatrix} +\begin{vmatrix} 2 & 5 \\ 1 & 1 \end{vmatrix} & -\begin{vmatrix} 3 & 5 \\ 2 & 1 \end{vmatrix} & +\begin{vmatrix} 3 & 2 \\ 2 & 1 \end{vmatrix} \\ -\begin{vmatrix} 1 & 5 \\ 1 & 1 \end{vmatrix} & +\begin{vmatrix} 1 & 5 \\ 2 & 1 \end{vmatrix} & -\begin{vmatrix} 1 & 1 \\ 2 & 1 \end{vmatrix} \\ +\begin{vmatrix} 1 & 5 \\ 2 & 5 \end{vmatrix} & -\begin{vmatrix} 1 & 5 \\ 3 & 5 \end{vmatrix} & +\begin{vmatrix} 1 & 1 \\ 3 & 2 \end{vmatrix} \end{bmatrix}$$

$$C = \begin{bmatrix} -3 & 7 & -1 \\ 4 & -9 & 1 \\ -5 & 10 & -1 \end{bmatrix}$$

$$Adj(A) = C^T$$

$$= \begin{bmatrix} -3 & 7 & -1 \\ 4 & -9 & 1 \\ -5 & 10 & -1 \end{bmatrix}^T$$

$$= \begin{bmatrix} -3 & 4 & -5 \\ 7 & -9 & 10 \\ -1 & 1 & -1 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} Adj(A)$$

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$$A^{-1} = \frac{1}{-1} \begin{bmatrix} -3 & 4 & -5 \\ 7 & -9 & 10 \\ -1 & 1 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & -4 & 5 \\ -7 & 9 & -10 \\ 1 & -1 & 1 \end{bmatrix}$$