KEMENTERIAN
PENDIDIKAN
MALAYSIA

QS 016

Mid-Semester Examination

Semester I

Session 2010/2011

Chow Choon Wooi

1. Solve $2log_3x = log_3 2 + log_3(3x - 4)$.

2. Given $(4 - i)x - 3y = 5 + i$. Find the value of x and y where $x, y \in \mathbb{R}$.

3. If $|x| \geq a$ is defined as $x \geq a \; or \; x \leq -a$, then solve $|2x| \geq x^2 - 3$.

4. Given α and β are roots of $x^2 + 3x + 1 = 0$. Find the quadratic equation with roots $\alpha + \frac{1}{\alpha}$ and $\beta + \frac{1}{\beta}$. **[Out of QS015 syllabus]**

5. The sum of the first 20 terms of an arithmetic sequence is 50, and the sum of the next 20 terms is -50. Find the first term and common difference of the sequence.

6. The function P is defined as $P(x) = px^3 + 2x^2 - qx + 2$ where $p, q \in \mathbb{R}$. P(x) is devisible by (x + 2) and 8 is the remainder when P(x) is divede by (x + 1).

   (a) Find the value of p and q. Hence, factorise P(x) completely.

   (b) Express $\frac{2x^2 - 6x+1}{P(x)}$ in partial fraction. **[Out of QS015 syllabus]**

7. (a) Expand $(4 + x)^{\frac{1}{2}}$ and $(1 + 3x)^{-1}$ in ascending powers of x up to ther term in x^2.

   (b) By using the result from (a), find the expansion of $\frac{(4+x)^{\frac{1}{2}}}{(1+3x)}$ in ascending powers of x up to the term in x^2. State the range of x for which the expansion is valid.

1. Solve $2log_3x = log_32 + log_3(3x - 4)$.

Solution

$2log_3x = log_32 + log_3(3x - 4)$

$log_3x^2 = log_32(3x - 4)$

$x^2 = 2(3x - 4)$

$x^2 = 6x - 8$

$x^2 - 6x + 8 = 0$

$(x - 4)(x - 2) = 0$

$x = 4, 2$

2. Given $(4-i)x-3y=5+i$. Find the value of x and y where $x, y \in \mathbb{R}$.

Solution

$(4-i)x-3y=5+i$

$4x-xi-3y=5+i$

$(4x-3y)-xi=5+i$

$-x\,i=i$ .... (1)

$x=-1$

$4x-3y=5$ .... (2)

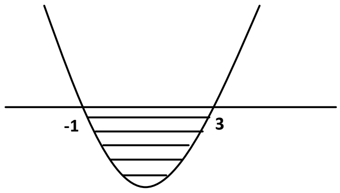
$4(-1)-3y=5$

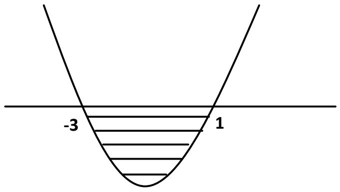
$y=-3$

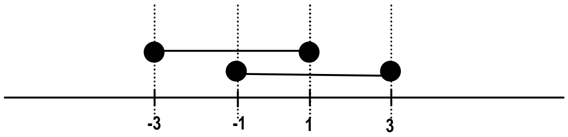
$\therefore x=-1, \quad y=-3$

3. If $|x| \geq a$ is defined as $x \geq a \text{ } or \text{ } x \leq -a$, then solve $|2x| \geq x^2 - 3$.

Solution

<table>
<tr><td colspan="3">$|2x| \geq x^2 - 3$</td></tr>
<tr><td>$2x \geq x^2 - 3$
$x^2 - 2x - 3 \leq 0$
$(x-3)(x+1) \leq 0$


[-1, 3]</td><td>or</td><td>$2x \leq -(x^2 - 3)$
$2x \leq -x^2 + 3$
$x^2 + 2x - 3 \leq 0$
$(x-1)(x+3) \leq 0$


[-3, 1]</td></tr>
<tr><td colspan="3">

$\therefore [-3, 3]$</td></tr>
</table>

4. Given α and β are roots of $x^2 + 3x + 1 = 0$. Find the quadratic equation with roots $\alpha + \frac{1}{\alpha}$ and $\beta + \frac{1}{\beta}$. **[Out of QS015 syllabus]**

Solution

$\alpha + \beta = -3, \qquad \alpha\beta = 1$

Form a new equation with roots:

$\alpha + \dfrac{1}{\alpha}, \qquad \beta + \dfrac{1}{\beta}$

Sum of new roots:

$$\alpha + \frac{1}{\alpha} + \beta + \frac{1}{\beta}$$

$$= \alpha + \beta + \frac{\alpha + \beta}{\alpha\beta}$$

$$= -3 - 3$$

$$= -6$$

Product of new roots:

$$\left(\alpha + \frac{1}{\alpha}\right)\left(\beta + \frac{1}{\beta}\right)$$

$$= \alpha\beta + \frac{\alpha}{\beta} + \frac{\beta}{\alpha} + \frac{1}{\alpha\beta}$$

$$= 1 + \frac{\alpha^2 + \beta^2}{\alpha\beta} + 1$$

$$= 2 + \frac{\alpha^2 + \beta^2}{\alpha\beta}$$

$$= 2 + \frac{\alpha^2 + \beta^2}{1}$$

$$= 2 + (\alpha^2 + \beta^2)$$

$$= 2 + (\alpha + \beta)^2 - \alpha\beta$$

$$= 2 + (-3)^2 - 2$$

$$= 9$$

The new equation: $x^2 + 6x + 9 = 0$

5. The sum of the first 20 terms of an arithmetic sequence is 50, and the sum of the next 20 terms is -50. Find the first term and common difference of the sequence.

Solution

$$S_n = \frac{n}{2}\,[2a + (n-1)d]$$

$$S_{20} = \frac{20}{2}\,[2a + (19)d] = 10\,[2a + 19d]$$

$$\frac{20}{2}\,[2a + (19)d] = 50$$

$2a + 19d = 5$ .... (1)

$$S_{40} = \frac{40}{2}\,[2a + (39)d] = 20\,[2a + 39d]$$

$$S_{40} - S_{20} = -50$$

$$20\,[2a + 39d] - 50 = -50$$

$40a + 780d = 0$ .... (2)

$Solve\ (1) and\ (2)$

$$d = -\frac{1}{4}$$

$$a = \frac{39}{8}$$

6. The function P is defined as $P(x) = px^3 + 2x^2 - qx + 2$ where $p, q \in \mathbb{R}$. P(x) is devisible by (x + 2) and 8 is the remainder when P(x) is divede by (x + 1).

(a) Find the value of p and q. Hence, factorise P(x) completely.

(b) Express $\frac{2x^2 - 6x+1}{P(x)}$ in partial fraction. **[Out of QS015 syllabus]**

Solution

(a) P(-2) = 0

-8p + 2q + 10 = 0 .... (1)

P(-1) = 8

-p + q + 4 = 8 .... (2)

Solve (1) and (2)

P = 3, q = 7

∴ P(x) = $3x^3 + 2x^2 - 7x + 2$

Using long division:

$$x+2\overline{\smash{\big)}\,3x^2 + 2x^2 - 7x + 2}\quad \text{quotient: } 3x^2 - 4x + 1$$

P(x) = $(x + 2)(3x^2 - 4x + 1)$

= (x + 2)(3x – 1)(x – 1)

(b) $\frac{2x^2-6x+1}{(x+2)(3x-1)(x-1)} = \frac{A}{(x+2)} + \frac{B}{(3x-1)} + \frac{C}{(x-1)}$

$2x^2 - 6x + 1 = A(3x-1)(x-1) + B(x+2)(x-1) + C(x+2)(3x-1)$

$When\ x = 1: \quad C = -\frac{1}{2}$

$When\ x = -2: \quad A = 1$

$When\ x = \frac{1}{3}: \quad B = \frac{1}{2}$

$\therefore A = 1;\ \ B = \frac{1}{2};\ \ C = -\frac{1}{2}$

(c) $\frac{2x^2-6x+1}{(x+2)(3x-1)(x-1)} = \frac{1}{(x+2)} + \frac{1}{2(3x-1)} - \frac{1}{2(x-1)}$

7. (a) Expand $(4+x)^{\frac{1}{2}}$ and $(1+3x)^{-1}$ in ascending powers of x up to ther term in x^2.

(b) By using the result from (a), find the expansion of $\frac{(4+x)^{\frac{1}{2}}}{(1+3x)}$ in ascending powers of x up to the term in x^2. State the range of x for which the expansion is valid.

Solution

(a) $(4+x)^{\frac{1}{2}}$

$$= 4^{\frac{1}{2}}\left(1+\frac{x}{4}\right)^{\frac{1}{2}}$$

$$= 2\left[1+\left(\frac{1}{2}\right)\left(\frac{x}{4}\right)+\frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)}{2!}\left(\frac{x}{4}\right)^2+\ \dots\right]$$

$$= 2\left[1+\frac{1}{8}x-\frac{1}{128}x^2+\ \dots\right]$$

$$= 2+\frac{1}{4}x-\frac{1}{64}x^2+\ \dots$$

$(1+3x)^{-1}$

$$= 1+\ (-1)(3x)+\frac{(-1)(-2)}{2!}(3x)^2+\ \dots$$

$$= 1-3x+9x^2+\cdots$$

(b) $\frac{(4+x)^{\frac{1}{2}}}{(1+3x)}$

$$= (4+x)^{\frac{1}{2}}\ (1+3x)^{-1}$$

$$= \left[2+\frac{1}{4}x-\frac{1}{64}x^2+\ \dots\right][1-3x+9x^2+\cdots]$$

$$= 2-6x+18x^2+\ \dots+\frac{1}{4}\ x-\frac{3}{4}\ x^2+\cdots-\frac{1}{64}x^2+\ \dots$$

$$= 2-\frac{23}{4}x+\frac{1103}{64}x^2$$

$$\left|\frac{x}{4}\right| < 1 \ \cap \ |x| < \frac{1}{3}$$

$$|x| < 4 \ \cap \ |x| < \frac{1}{3}$$

$$|x| < 4 \ \cap \ |x| < \frac{1}{3}$$

Therefore:

$|x| < \frac{1}{3}$

$$-\frac{1}{3} < x < \frac{1}{3}$$