KEMENTERIAN PENDIDIKAN MALAYSIA

# QS 015

# Mid-Semester Examination

# Semester I

# Session 2012/2013

**Chow Choon Wooi**

1. Given that $(x + yi)(1 + i) = 1 + 2i$, find the values of x and y.

2. Solve $3\,log_8 x - log_x 64 + 1 = 0$.

3. Solve the equation $4^{2x+1} - 65(4^x) + 16 = 0$.

4. Solve the following inequalities.

   (a) $|2x - 3| > 5$

   (b) $\frac{1}{x+1} \leq \frac{3}{x-1}$

5. (a) A geometric sequence has seven terms. The first and the fourth terms are 8 and 216 respectively. Find the common ratio and the last term of the sequence.

   (b) Show that the $(r+1)^{th}$ term of binomial expansion $\left(x^2 + \frac{1}{x}\right)^{10}$ can be written as $T_{r+1} = \binom{10}{r} x^{20-3r}$. Hence, find the coefficient of $x^2$.

6. Given the matrix $B = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 2 \\ 1 & 2 & 1 \end{bmatrix}$.

   (a) Find

   (i) $|B|$

   (ii) adjoint B

   (iii) $B^{-1}$

   (b) Hence, solve the following system of linear equations.

   X + 2y = 2

   Y + 2z = 3

   X + 2y + z = 1

1. Given that $(x + yi)(1 + i) = 1 + 2i$, find the values of x and y.

***Solution***

$$(x + yi)(1 + i) = 1 + 2i$$

$$x + xi + yi - y = 1 + 2i$$

$$x - y + (x + y)i = 1 + 2i$$

$$x - y = 1$$

$$x + y = 2$$

$$\therefore x = \frac{3}{2}, \quad y = \frac{1}{2}$$

2. Solve $3\,log_8 x - log_x 64 + 1 = 0$.

***Solution***

$$3\,log_8 x - log_x 64 + 1 = 0$$

$$3\,log_8 x - \frac{log_8 64}{log_8 x} + 1 = 0$$

Let $u = log_8 x$

$$3u - \frac{2}{u} + 1 = 0$$

$$3u^2 - 2 + u = 0$$

$$3u^2 + u - 2 = 0$$

$$(3u - 2)(u + 1) = 0$$

$u = \frac{2}{3}$ or $u = -1$

$log_8 x = \frac{2}{3}$ or $log_8 x = -1$

$x = 8^{\frac{2}{3}} = 4$ or $x = 8^{-1} = \frac{1}{8}$

3. Solve the equation $4^{2x+1} - 65(4^x) + 16 = 0$.

***Solution***

$4^{2x+1} - 65(4^x) + 16 = 0$

$4(4^x)^2 - 65(4^x) + 16 = 0$

Let $u = 4^x$

$4u^2 - 65u + 16 = 0$

$(4u - 1)(u - 16) = 0$

$4u = 1$ or $u = 16$

$u = \frac{1}{4}$ or $4^x = 4^2$

$4^x = 4^{-1}$ or $x = 2$

$x = -1$

$\therefore x = -1$ or $x = 2$

4. Solve the following inequalities.

(a) $|2x-3|>5$

(b) $\frac{1}{x+1} \leq \frac{3}{x-1}$

***Solution***

(a) $|2x-3|>5$

$2x-3>5$ or $2x-3<-5$

$2x>8$ or $2x<-2$

$x>4$ or $x<-1$

$\therefore \{x: x<-1 \cup x>4\}$

(b) $\frac{1}{x+1} \leq \frac{3}{x-1}$

$$\frac{1}{x+1}-\frac{3}{x-1} \leq 0$$

$$\frac{(x-1)-3(x+1)}{(x+1)(x-1)} \leq 0$$

$$\frac{-2x-4}{(x+1)(x-1)} \leq 0$$

$$\frac{-2(x+2)}{(x+1)(x-1)} \leq 0$$

$$\frac{2(x+2)}{(x+1)(x-1)} \geq 0$$

$x=-2$ $x=-1$ $x=1$

| | $(-\infty,-2)$ | $(-2,-1)$ | $(-1,1)$ | $(1,\infty)$ |
|---|---|---|---|---|
| $(x+2)$ | - | + | + | + |
| $(x+1)$ | - | - | + | + |
| $(x-1)$ | - | - | - | + |
| $\frac{(x+2)}{(x+1)(x-1)}$ | - | (+) | - | (+) |

$\therefore [-2,-1) \cup (1,\infty)$

5. (a) A geometric sequence has seven terms. The first and the fourth terms are 8 and 216 respectively. Find the common ratio and the last term of the sequence.

(b) Show that the $(r+1)^{th}$ term of binomial expansion $\left(x^2+\frac{1}{x}\right)^{10}$ can be written as $T_{r+1}=\binom{10}{r}x^{20-3r}$. Hence, find the coefficient of $x^2$.

***Solution***

(a) $a = 8,\ T_4 = 216$

$8r^3 = 216$

$r^3 = 27$

$r = 3$

$T_7 = 8(3)^6$

$= 5832$

(b) $T_{r+1} = \binom{10}{r}(x^2)^{10-r}\left(\frac{1}{x}\right)^r$

$= \binom{10}{r}(x^{20-2r})(1)^r(x)^{-r}$

$= \binom{10}{r}(x^{20-3r})$

$x^{20-3r} = x^2$

$20-3r = 2$

$r = 6$

The coefficient, $x^2 = \binom{10}{6} = 210$

6. Given the matrix $B = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 2 \\ 1 & 2 & 1 \end{bmatrix}$.

(a) Find

(i) $|B|$

(ii) adjoint B

(iii) $B^{-1}$

(b) Hence, solve the following system of linear equations.

X + 2y = 2

Y + 2z = 3

X + 2y + z = 1

***<u>Solution</u>***

(a) $B = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 2 \\ 1 & 2 & 1 \end{bmatrix}$

i. $|B| = 1\begin{vmatrix} 1 & 2 \\ 2 & 1 \end{vmatrix} - 2\begin{vmatrix} 0 & 2 \\ 1 & 1 \end{vmatrix} + 0\begin{vmatrix} 0 & 1 \\ 1 & 2 \end{vmatrix}$

= (1 – 4) – 2(0 – 2) + 0

= 1

ii. $$Adj\ (B) = \begin{bmatrix} +\begin{vmatrix} 1 & 2 \\ 2 & 1 \end{vmatrix} & -\begin{vmatrix} 0 & 2 \\ 1 & 1 \end{vmatrix} & +\begin{vmatrix} 0 & 1 \\ 1 & 2 \end{vmatrix} \\ -\begin{vmatrix} 2 & 0 \\ 2 & 1 \end{vmatrix} & +\begin{vmatrix} 1 & 0 \\ 1 & 1 \end{vmatrix} & -\begin{vmatrix} 1 & 2 \\ 1 & 2 \end{vmatrix} \\ +\begin{vmatrix} 2 & 0 \\ 1 & 2 \end{vmatrix} & -\begin{vmatrix} 1 & 0 \\ 0 & 2 \end{vmatrix} & +\begin{vmatrix} 1 & 2 \\ 0 & 1 \end{vmatrix} \end{bmatrix}^T$$

$$= \begin{bmatrix} -3 & 2 & -1 \\ -2 & 1 & 0 \\ 4 & -2 & 1 \end{bmatrix}^T$$

$$= \begin{bmatrix} -3 & -2 & 4 \\ 2 & 1 & -2 \\ -1 & 0 & 1 \end{bmatrix}$$

*iii.* $B^{-1} = \frac{1}{|B|} Adj(B)$

$$= \frac{1}{1}\begin{bmatrix} -3 & -2 & 4 \\ 2 & 1 & -2 \\ -1 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -3 & -2 & 4 \\ 2 & 1 & -2 \\ -1 & 0 & 1 \end{bmatrix}$$

(b) $\begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 2 \\ 1 & 2 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix}$

$BX = C$

$B^{-1}BX = B^{-1}C$

$X = B^{-1}C$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -3 & -2 & 4 \\ 2 & 1 & -2 \\ -1 & 0 & 1 \end{bmatrix}\begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -8 \\ 5 \\ -1 \end{bmatrix}$$

$\therefore x = -8, \quad y = 5, \quad z = -1$